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Physical Sciences

Review article

The Scalar Field Model of Dark Energy in the Framework of the Holographic Principle

El modelo del campo escalar de la energía oscura en el marco del principio holográfico

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Abstract

We study cosmological solutions for a scalar field minimally coupled to the curvature, in the framework of holographic dark energy. Phantom solutions can be obtained without introducing ghosts' degrees of freedom, and the autonomous system contains stable accelerated expansion solutions and de Sitter attractors. For the non-minimally coupled scalar field the special case of the conformal coupling is analyzed, and it is shown that dynamically evolving scalar field produces the effect of the cosmological constant.

Keywords: Dark Energy; Holographic Principle; Scalar Field

Resumen

Se estudian soluciones cosmológicas para un campo escalar acoplado mínimamente a la curvatura, en el marco del principio holográfico. Se pueden obtener soluciones phantom sin introducir grados de libertad fantasma, y el sistema autónomo contiene soluciones de expansión acelerada estables y atractores de Sitter. Para el campo con acoplamiento no-mínimo se analiza el caso especial del acoplamiento conforme y se demuestra que un campo escalar que evoluciona dinámicamente puede producir el efecto de la constante cosmológica.

Palabras Clave: Energía Oscura; Principio Holográfico; Campo Escalar

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Introduction

The current accelerated expansion of the universe (**Riess, *et al.,*** 1998), (**Perlmutter*et al.,*** 1998), (**Kowalski, *et al.,*** 2008), (**Hicken, *et al.,*** 2009), (**Ade, *et al.,*** 2016), supposes a great challenge for the contemporary science. The source of this expansion, called dark energy, may consist of cosmological constant, conventionally associated with the energy of the vacuum or alternatively, could came from a dynamical varying scalar field at late times which also account for the missing energy density in the universe. In order to avoid the fine tuning and the coincidence problems, which relate to the inflationary behavior of the early universe and the late time dark energy dominated regime, the dark energy should have dynamical nature. This stimulates the interest in scalar fields that naturally arise in particle physics, including string theory, supergravity and generalized gravity theories such as the scalar tensor theories of gravity. So far a wide variety of scalar field models of dark energy have been proposed, including quintessence (**Ratra and Peebles,** 1988), k-essence (**Armendariz, *et al.,*** 2000), tachyon (**Padmanabhan and Choudhury,** 2002), phantom (**Caldwell,** 2002), ghost condensate (**Arkani-Hamed, *et al.,*** 2004) among others. The quintessence is an ordinary scalar field minimally coupled to gravity, with

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particular potentials that lead to late time accelerated expansion. The equation of state for spatially homogeneous scalar field satisfies the inequality $w < -1$, and therefore can produce accelerated expansion. According to the current observational data (**Riess, *et al.,*** 1998), (**Perlmutter, *et al.,*** 1998), (**Kowalski, *et al.,*** 2008), (**Hicken, *et al.,*** 2009) , (**Ade, *et al.,*** 2016), the dark energy equation of state could be in a narrow region below the cosmological constant divide $w = -1$, i. e. $w < -1$, indicating that accelerated expansion is going through a phantom phase. Therefore, the quintessence field may not be adequate to describe the state of accelerated expansion of the universe and models which allow the phantom phase seem more suitable.

Among the models of dark energy, especially interesting is the scalar field model non-minimally coupled to curvature, which normally arises in quantum field theory in curved space time (**Ford,** 19987, **Birrell and Davis,** 1982) or after compactification of higher dimensional gravity theories and in the context of string theories. These kinds of couplings have been proposed by many authors to address the dark energy problem since these couplings provide in principle a mechanism to evade the coincidence problem, allow phantom crossing in some cases (**Perivolaropoulos,** 2005). A dynamical system for non-minimally coupled scalar field was studied in (**Sami, *et al.,*** 2012), and in (**Granda and Jimenez,** 2017, **Granda and Jimenez,** 2018), the autonomous system analysis was studied for models with non-minimal Gauss-Bonnet and non-minimal kinetic couplings respectively.

Another interesting approach to explain the nature of the dark energy is based on some facts of quantum gravity known as holographic principle (**'t Hooft,** 1993), (**Susskind,** 1994), (**Bousso,** 1999). This principle establishes a connection between the short distance (ultraviolet) cut-off and the long distance (infrared) cut-off, given by a restriction on the size of the system in such a manner that prevents the formation of black holes with size larger than the size of the system (**Cohen, *et al.,*** 1999), (**Hsu,** 2004). Applied to the dark energy issue, if we take the whole universe into account, then the vacuum energy related to this holographic principle is viewed as dark energy, usually called holographic dark energy. Based on this principle, the proposal of holographic dark energy have been developed in (**Cohen, *et al.,*** 1999), (**Hsu,** 2004), (**Li,** 2004). Different IR cut-off scales such as Hubble scale, particle horizon and event horizon have been proposed to establish the holographic dark energy models. The Hubble scale can not give rise to an accelerated universe (**Hsu,** 2004), while the event horizon can produce an accelerated expansion (**Li,** 2004), but has problem with causality. A Holographic density model that is free of causality problem can explain the coincidence problem was presented in (**Granda and Oliveros,** 2008), (**Granda and Oliveros,** 2008a).

In this work we consider the scalar field in the framework of the holographic principle as proposed in (**Granda and Oliveros,** 2008), with the holographic density as the background vacuum energy and analyze the possible accelerating regimes that could take place under the effect of a scalar field in the framework the holographic principle. The non-minimal coupling in the framework of holographic principle has been studied in (**Ito,** 2005), (**Setare and Saridakis,** 2008). It should be said that such extension of the scalar field in the frame of holographic principle needs further theoretical foundation related with the microscopic nature of the vacuum energy.

The scalar field in the framework of the holographic principle

Let us consider the simplest model of non-minimally coupled scalar field with potential in the presence of holographic dark energy (vacuum energy). The action for the scalar field with matter in a general background is given by

$$S = \int d^4 x \sqrt{-g} \left[\frac{1}{2\varkappa^2} R - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - V(\varphi)\right] + S_m \qquad (1)$$

where $\varkappa^2 = 8\pi G$, S_m is the action for the matter fields which in the present case includes the usual baryonic and dark matter. Initially we consider the limit of dark energy dominance and neglect the baryonic and dark matter contribution. We are considering the flat Friedmann-Robertson-Walker (FRW) metric with signature (-,+,+,+). The Friedmann equation including the energy contribution from the holographic principle [40] is

$$H^2 = \frac{\kappa^2}{3}\left(\frac{1}{2}\dot{\varphi}^2 + V(\varphi) + \rho_\Lambda\right), \qquad (2)$$

where

$$\rho_\Lambda = \frac{3}{\kappa^2}(\alpha H^2 + \beta \dot{H}), \qquad (3)$$

and ρ_Λ is the energy contribution from the holographic principle. The equation of motion for the scalar field is given by

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{dV}{d\varphi} = 0, \qquad (4)$$

and the effective equation of state resulting from the time evolution of this model is

$$w_{eff} = -1 - \frac{2\dot{H}}{3H^2} \quad (5)$$

Quintessence Solutions

To solve the equations (2) and (4), we can propose the following functions

$$H = \frac{p}{t}, \quad \varphi = \varphi_0 \ln\frac{t}{t_0}, \quad V = V_0\, e^{\lambda\varphi}. \qquad (6)$$

Replacing these expressions in (2) it is found that under the restriction

$$\lambda\varphi_0 = -2,$$

the following relation takes place (here we use $\varkappa^2 = M_p^2$)

$$3M_p^2\,[(1 - \alpha)\,p^2 + \beta p] = \frac{1}{2}\,\varphi_0^2 + t_0^2\,V_0 \qquad (7)$$

And from the equation of motion (4) one finds

$$(3p - 1)\,\varphi_0^2 - 2t_0^2\,V_0 = 0. \qquad (8)$$

The last two equations give us the initial values and in terms of the parameters

$$\varphi_0^2 = 2M_p^2\,[(1 - \alpha)\,p + \beta], \qquad (9)$$

and

$$t_0^2\,V_0 = M_p^2\,(3p - 1)\,[(1 - \alpha)\,p + \beta]. \qquad (10)$$

The scalar field and the potential keep the same functional dependence on time as in the simplest case of the canonical scalar field on FRW background with the advantage, in the present case, that there exist phantom solutions (leading to BR singularities) without resorting to ghost degrees of freedom. Using the initial value φ_0 one finds the power p as

$$p = \frac{1}{\alpha - 1}\left(\beta - \frac{\varphi_0^2}{2Mp^2}\right). \qquad (11)$$

In absence of scalar field this equation gives the usual restriction for accelerated expansion [40], $\beta > \alpha - 1$, and with the scalar field the Eq. (11) gives more possibilities including negative values of p leading to super acceleration, depending on the relation between φ_0 and Mp. Thus, the conditions for accelerated expansion ($p > 1$) take the form

$$\beta < \frac{\gamma^2}{2} \quad \textit{and} \quad \frac{1}{2}\,(2 + 2\beta - \gamma^2) < \alpha < 1 \quad (12)$$

or

$$\beta > \frac{\gamma^2}{2} \quad \textit{and} \quad 1 < \alpha < \frac{1}{2}(2 + 2\beta - \gamma^2) \quad (13)$$

where $\gamma = \varphi_0/Mp$. Note that these conditions take place whether γ is greater than or less than 1.

Big Rip solutions

Let's consider the following solutions

$$H = \frac{q}{t_c - t}, \quad \varphi = \varphi_0 \ln(t_c - t), \quad V = V_0\, e^{\lambda\varphi} \qquad (14)$$

with q > 0, which lead to Big Rip singularity at $t = t_c$. Replacing (14) in the Eqs. (2) and (4) one finds the same restriction, $\lambda = -2/\varphi_0$, and the relations

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$$\varphi_0^2 = 6M_p^2 \frac{(1-\alpha)q^2 - \beta q}{2+3q}, \qquad (15)$$

and

$$\varphi_0^2 = 6M_p^2 \frac{(1-\alpha)q^2 - \beta q}{2+3q}, \qquad (16)$$

and the expression for q in terms of φ_0 is given by

$$q = \frac{1}{12(1-\alpha)}\left[3(2\beta+\gamma^2) + \sqrt{3(2\beta+\gamma^2)^2 + 48(\alpha-1)\gamma^2}\right] \quad (17)$$

where $\gamma = \varphi_0/Mp$. The effective equation of state from (5) takes the value

$$w_{eff} = -1 - \frac{1}{q}.$$

From (15) and (16) follows that, given $q > 0$, we don't need to resort to ghost degrees of freedom, since $\varphi_0^2 > 0$ whenever $(1 - \alpha)\, q - \beta > 0$, which implies that $V_0 > 0$. Thus, the standard canonical scalar field in the vacuum background generated by the holographic principle can describe an accelerated expansion with EoS bellow the phantom divide, i.e. $w_{eff} < -1$.

The matter contribution and critical points

In this case the cosmological equations take the form

$$H^2 = \frac{\kappa^2}{3}\left(\frac{1}{2}\dot{\varphi}^2 + V(\varphi) + \rho_m\right) + \alpha H^2 + \beta \dot{H}, \qquad (18)$$

$$-2\dot{H} - 3H^2 = \kappa^2\left(\frac{1}{2}\dot{\varphi}^2 - V(\varphi) + p_m + p_\Lambda\right), \qquad (19)$$

with the scalar potential given by

$$V = V_0 e^{\lambda\varphi} \qquad (20)$$

where ρ_m and ρ_m are the density and pressure of the matter contribution respectively (baryonic and dark matter), p_Λ is the pressure corresponding to the vacuum energy. Both type of energy contributions are modeled by ideal fluids that obey the independent continuity equations

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0, \quad \dot{\rho}_\Lambda + 3H(\rho_\Lambda + p_\Lambda) = 0 \qquad (21)$$

additionally, we have the equation of motion for the scalar field given by (4) which is not independent of (18) and (19). Here we assume that the equation of state for matter, $w_m = \rho_m/\rho_m$ is constant, which allows the integration of the conservation equation, giving $\rho_m = \rho_{m0}\, a^{-3(1+w_m)}$. To understand some dynamical properties of the model we will consider the autonomous system and analyze the properties of the critical points. The dynamical variables can be deduced from (18) and will be defined as

$$x = \frac{\kappa\dot{\varphi}}{\sqrt{6}H}, \quad y = \frac{\kappa\sqrt{V}}{\sqrt{3}H}, \quad \Omega_m = \frac{\kappa^2\rho}{3H^2}, \quad \Omega_\Lambda = \alpha + \beta\frac{\dot{H}}{H^2} \qquad (22)$$

which lead to the following restriction from Eq. (18)

$$x^2 + y^2 + \Omega_m + \Omega_\Lambda = 1 \qquad (23)$$

and from Eqs. (4), (19) and (21), it is straightforward to derive the following equations

$$\frac{dx}{dN} = -3x + \frac{\sqrt{6}}{2}\lambda y^2 + \frac{3x}{2-3\gamma\beta}\left((2-\gamma)x^2 + \gamma(1-y^2) - \alpha\gamma\right) \qquad (24)$$

$$\frac{dy}{dN} = -\frac{\sqrt{6}}{2}\lambda xy + \frac{3y}{2-3\gamma\beta}\left((2-\gamma)x^2 + \gamma(1-y^2) - \alpha\gamma\right) \qquad (25)$$

where $N = ln\ a$ is the slow-roll variable and $\gamma = 1 + w_m$. Note that if one sets $\alpha = \beta = 0$, then the Eqs. (21)-(25) reduce to the autonomous system for the uncoupled quintessence scalar [33]. The effective equation of state is given by

$$w_{eff} = -1 + \frac{2[(2-\gamma)x^2 + \gamma(1-y^2) - \alpha\gamma]}{2-3\gamma\beta} \qquad (26)$$

The following are the fixed points of the dynamical system (23)-(25).

$$
\begin{aligned}
&P1=(x_c,y_c)=(0,0), \qquad P2=\left(-\sqrt{\frac{2+\gamma(\alpha-3\beta-1)}{2-\gamma}},0\right),\\
&P3=\left(\sqrt{\frac{2+\gamma(\alpha-3\beta-1)}{2-\gamma}},0\right),\\
&P4=\left(\frac{3\gamma(2-\beta\lambda^2)+2\lambda^2-D_1}{4\sqrt{6}\gamma},-\frac{\sqrt{[2(\lambda^2-6)+6\gamma-3\beta\gamma\lambda^2]D_1-D_2}}{4\sqrt{3}\gamma}\right),\\
&P5=\left(\frac{3\gamma(2-\beta\lambda^2)+2\lambda^2-D_1}{4\sqrt{6}\gamma},\frac{\sqrt{[2(\lambda^2-6)+6\gamma-3\beta\gamma\lambda^2]D_1-D_2}}{4\sqrt{3}\gamma}\right),\\
&P6=\left(\frac{3\gamma(2-\beta\lambda^2)+2\lambda^2+D_1}{4\sqrt{6}\gamma},-\frac{\sqrt{[2(\lambda^2-6)+6\gamma-3\beta\gamma\lambda^2]D_1-D_2}}{4\sqrt{3}\gamma}\right),\\
&P7=\left(\frac{3\gamma(2-\beta\lambda^2)+2\lambda^2+D_1}{4\sqrt{6}\gamma},\frac{\sqrt{[2(\lambda^2-6)+6\gamma-3\beta\gamma\lambda^2]D_1-D_2}}{4\sqrt{3}\gamma}\right),
\end{aligned}
\tag{27}
$$

where

$$
\begin{aligned}
&D_1=\sqrt{9(\beta\lambda^2-2)^2\gamma^2-12\lambda^2(\beta\lambda^2-4\alpha+2)\gamma+4\lambda^4}\\
&9(\beta\lambda^2-2)^2\gamma^2-3\gamma(24-8\alpha\lambda^2+4\beta\lambda^4-12\beta\lambda^2)+4\lambda^2(\lambda^2-6).
\end{aligned}
\tag{28}
$$

By setting $\alpha=\beta=0$ we recover the critical points corresponding to the minimally coupled scalar field. The critical points characterize different cosmological scenarios, depending on the parameters α, β, γ, λ. Here we illustrate some cases that involve the effects of the holographic density:

The point P1: $w_{eff}=-1+\frac{2(\alpha-1)\gamma}{3\beta\gamma-2}$, $\Omega_m=\frac{2(\alpha-1)}{3\beta\gamma-2}$, $\Omega_\varphi=0$, $\Omega_\Lambda=\frac{2\alpha-3\beta\gamma}{2-3\beta\gamma}$.

The corresponding eigenvalues are

$$\left(\frac{3(\alpha-1)\gamma}{3\beta\gamma-2},\frac{3(\alpha-3\beta-1)\gamma+6}{3\beta\gamma-2}\right).$$

If we take $\alpha=1$, then $w_{eff}=-1$ and $\Omega_m=0$. Then the energy becomes dominated by the holographic component and the point is a de Sitter solution with eigenvalues $(0,-3)$, which corresponds to marginally stable fixed point. On the other hand, if we take $\alpha=3\beta\gamma/2$, then $w_{eff}=\gamma-1=w_m$ with the energy density dominated by matter $\Omega_m=1$, and eigenvalues

$$\left(\frac{3\gamma}{2},\frac{3}{2}(\gamma-2)\right),$$

which corresponds to a saddle point for $0<\gamma<2$. The case $\alpha=\beta=0$, which corresponds to the standad quintessence scalar, gives the known matter dominated solution ($\Omega_m=1$) with $w_{eff}=w_m$ [33]. In this last case the eigenvalues become $(\frac{3}{2}(\gamma-2),\frac{3\gamma}{2})$ giving a saddle point for $1<\gamma<2$. The de Sitter solution in this point is due to the holographic component.

The point P2: $w_{eff}=1$, $\Omega_m=\frac{2(\alpha-3\beta)}{\gamma-2}$, $\Omega_\varphi=\frac{3\beta\gamma-\alpha\gamma+\gamma-2}{\gamma-2}$, $\Omega_\Lambda=\alpha-3\beta$.

The eigenvalues are

$$\left(\frac{6(3\beta\gamma-\alpha\gamma+\gamma-2)}{3\beta\gamma-2},3+\sqrt{\frac{3(3\beta\gamma-\alpha\gamma+\gamma-2)}{2(\gamma-2)}}\lambda\right).$$

If we take $\alpha=3\beta$, then $\Omega_m=0$ and the solution becomes dominated by the scalar field ($\Omega_\varphi=1$) and is stable only under the conditions $\lambda<-\sqrt{6},\beta>\frac{2}{3\gamma},\ 0<\gamma<2$. Note that $w_{eff}=1$ for any value of the parameters, including the case of the quintessence scalar field corresponding to $\alpha=\beta=0$ ($P2=(-1,0)$, giving also $\Omega_m=0$), with the eigenvalues given by $(-3(\Upsilon-2),3+\sqrt{\frac{3}{2}}\lambda)$, which lead to unstable ($\lambda>-6$) or saddle point ($\lambda<-6$) for $0<\gamma<2$. The difference between these two cases is that the inclusion of the holographic component can lead to stable fixed point. There is also a stable solution dominated by matter ($\Omega_m=1$, $\Omega_\varphi+\Omega_\Lambda=0$) if one sets

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$$\gamma = 2(\alpha - 3\beta + 1) \text{ and } \lambda < -\sqrt{6},\ \alpha > \frac{\lambda^4 - 6\lambda^2 + 36}{\lambda^4 - 6\lambda^2} \text{ and}$$

$$\frac{\alpha\lambda^2 + 6}{3\lambda^2} < \beta < \frac{\alpha + 1}{6} + \frac{1}{6}\sqrt{\alpha^2 + 2\alpha - 3}.$$

In this case the scalar field and holographic contribution cancel each other, which is not of interest since one of these density parameters should be negative.

The point P3: his point gives the same results

$$w_{eff} = 1,\quad \Omega_m = \frac{2(\alpha - 3\beta)}{\gamma - 2},\quad \Omega_\varphi = \frac{3\beta\gamma - \alpha\gamma + \gamma - 2}{\gamma - 2},\quad \Omega_\Lambda = \alpha - 3\beta,$$

with eigenvalues

$$\left(\frac{6(3\beta\gamma - \alpha\gamma + \gamma - 2)}{3\beta\gamma - 2}, 3 - \sqrt{\frac{3(3\beta\gamma - \alpha\gamma + \gamma - 2)}{2(\gamma - 2)}}\lambda\right),$$

and the stability for the scalar field dominated solution is reached under the conditions $\lambda > \sqrt{6}, \beta > \frac{2}{3\gamma},\ 0 < \gamma < 2$. The quintessence case ($\alpha = \beta = 0$), giving P3=(1,0)) is unstable for $\lambda < \sqrt{6}$ and saddle for $\lambda > \sqrt{6}$, given that $0 < \gamma < 2$. The matter dominated solution that takes place for $\gamma = 2\ (\alpha - 3\beta + 1)$, is stable under the restrictions

$$> \sqrt{6},\ \alpha > \frac{\lambda^4 - 6\lambda^2 + 36}{\lambda^4 - 6\lambda^2} \text{ and } \frac{\alpha\lambda^2 + 6}{3\lambda^2} < \beta < \frac{\alpha + 1}{6} + \frac{1}{6}\sqrt{\alpha^2 + 2\alpha - 3}.$$

The point P4: First we note that for the case , and independently of the other parameters (β, γ, λ), $w_{eff} = -1$ and $\Omega_m = 0$, $\Omega_\varphi = 0$, $\Omega_\Lambda = 1$. Due to its complexity to calculate the eigenvalues, we analyzed the case $\gamma = 1$, and have found that the point is an attractor if $\beta > \frac{2(\lambda^2 + 3)}{3\lambda^2}$ for any $\lambda \neq 0$. This de Sitter attractor is due exclusively to the holographic component. For the quintessence scalar field $\alpha = \beta = 0$, the fixed point takes the values P4 $= \left(\sqrt{\frac{3}{2}}\frac{\lambda}{\gamma}, -\sqrt{\frac{3\gamma(2-\gamma)}{2\lambda^2}}\right)$ giving the stable scaling solution with $w_{eff} = w_m$ for $\lambda^2 > 3\gamma$. One case of quintessence solution takes place if one sets $\alpha = \beta = 2/\lambda^2$. In this case we find $w_{eff} = \frac{1}{6}(\lambda^2 - 6 - \sqrt{\lambda^4 - 12\gamma(\lambda^2 - 2)})$ with $\Omega_m = 1 - 2/\lambda^2$, $\Omega_\varphi = \frac{1}{2} - \frac{\sqrt{\lambda^4 - 12\gamma(\lambda^2 - 2)}}{2\lambda^2}$, $\Omega_\Lambda = \frac{1}{2} - \frac{4 - \lambda^2 + \sqrt{\lambda^4 - 12\gamma(\lambda^2 - 2)}}{2\lambda^2}$. This point is an attractor with $0 < \Omega_m, \Omega_\varphi, \Omega_\Lambda < 1$ and $-1 < w_{eff} < 0$ if $-2\sqrt{\frac{2}{3}} < \lambda < -\sqrt{2}$ and $0 < \gamma < \lambda^2/3$. Another stable quintessence solution is obtained by taking $\alpha = \gamma/3$ and $\beta = 2/\lambda^2$, giving $w_{eff} = \frac{1}{6}(\lambda^2 - 6 - \sqrt{\lambda^2(\lambda^2 + 4\gamma^2 - 12\gamma)})$. This point is an attractor with

$$-1 < w_{eff} < 1,\ \Omega_m = 1 - \frac{\gamma}{3},\ \Omega_\varphi = \frac{1}{2} - \frac{\sqrt{\lambda^2 + 4\gamma^2 - 12\gamma}}{2\lambda},$$

$$\Omega_\Lambda = \frac{2\gamma - 3}{6} + \frac{\sqrt{\lambda^2 + 4\gamma^2 - 12\gamma}}{2\lambda},$$

where all the density parameters are in the interval $0 < \Omega_m, \Omega_\varphi, \Omega_\Lambda < 1$ whenever $\frac{3\sqrt{2}}{2} < \lambda \leq 3$ and $\frac{\sqrt{9 - \lambda^2}}{2} + \frac{3}{2} < \gamma < \lambda^2/3$ or $\frac{11}{4} < \lambda < 3$ and $\frac{\sqrt{3(48 - 5\lambda^2)}}{2\lambda^2} < \gamma < \frac{\lambda^2}{3}$.

The point **P5** as in the case of the point **P4** contains a de Sitter attractor ($w_{eff} = -1$) dominated by the holographic component ($\Omega_\Lambda = 1$) for $\alpha = 1$, and the stability results from the restriction $\gamma = 1$ and $\beta > 2/3\ (\lambda^2 + 3)$ for any $\lambda \neq 0$. This critical point contains also the quintessence solutions described for the point **P4** with the same stability properties.

The point P6: For the scalar quintessence field, taking $\alpha = \beta = 0$, this critical point takes the values P6 $= \left(\frac{\lambda}{\sqrt{6}}, \frac{\sqrt{6 - \lambda^2}}{\sqrt{6}}\right)$ and becomes stable node dominated by the scalar field with $w_{eff} = -1 + \lambda^2/3$, provided that $\lambda < \sqrt{6}$ and $\gamma > \lambda^2/3$. By setting $\alpha = 1$ in this point we find

$$w_{eff} = -1 + \gamma + \left(\frac{1}{3} - \frac{1}{2}\beta\gamma\right)\lambda^2,\ \Omega_m = \frac{\beta\lambda^2(4\lambda^2 - 6\gamma(\beta\lambda^2 - 2))}{8\lambda^2},$$

$$\Omega_\varphi = 1 - \frac{3\beta\gamma}{2} + \frac{3\gamma}{\lambda^2},\ \Omega_\Lambda = \frac{1}{4}(3\beta\gamma - 2)(\beta\lambda^2 - 2).$$

This point satisfies all the conditions for an attractor solution with accelerated expansion, but the analytical expression for the conditions on the parameters are very large and we limit ourselves here to the two numerical examples: taking $\lambda = \sqrt{6}$, $\beta = 0.9$, $\gamma = 1.05$, it is found $w_{eff} = -0.785$, $\Omega_m = 0.18$, $\Omega_\varphi = 0.11$ and $\Omega_\Lambda = 0.71$. Taking $\lambda = 3\sqrt{3}$,

$\beta = 0.7$, $\gamma = 1.065$, one finds $w_{eff} = -0.999$, $\Omega_m = 0.0007$, $\Omega_\varphi = 0.00008$ and $\Omega_\Lambda = 0.999$. The above expressions for the main physical parameters simplify if one sets $\beta = 2/\lambda^2$. In this case we obtain

$$w_{eff} = -1 + \lambda^2/3,\ \Omega_m = 0,\ \Omega_\varphi = 1,\ \Omega_\Lambda = 0.$$

This scalar field dominated critical point is an attractor if

$$0 < \lambda < \sqrt{2} \text{ and } \gamma \geq \frac{36\lambda^2+12\lambda^4+\lambda^6}{108(1-\lambda^2)+27\lambda^4}.$$

By taking $\alpha = 1$ and $\beta = \frac{2(3\gamma+\lambda^2)}{3\gamma\lambda^2}$, this point gives a de Sitter saddle point dominated by the holographic component ($\Omega_\Lambda = 1$, $\Omega_m = 0$, $\Omega_\varphi = 0$) with eigenvalues (-3,0).

The point **P7** presents the same properties as the point **P6**, leading to stable de Sitter and quintessence attractors.

Non-minimal coupling in the framework of the holographic principle

Here we assume the generalization of the holographic principle in the presence of non-minimally coupled scalar field. Let us start with the following action for a dark energy dominated universe

$$S = \int d^4x\sqrt{-g}\,[\frac{1}{2\kappa^2}R - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}\xi R\varphi^2], \tag{29}$$

Variation with respect to the metric, and assuming that the scalar field φ has only time dependence, gives the following modified Friedman Eqs. in the flat FRW background [44, 45].

$$H^2 = \frac{\kappa^2}{3}(\frac{1}{2}\dot{\varphi}^2 + 6\xi H\varphi\dot{\varphi} + 3\xi H^2\varphi^2 + \rho_\Lambda), \tag{30}$$

which corresponds to the (00) component of the variation with respect to the metric, and for the (11) component it is obtained

$$-2\dot{H} - 3H^2 = \kappa^2\,[(\frac{1}{2} - 2\xi)\,\dot{\varphi}^2 - 2\xi(\varphi\ddot{\varphi} + 2H\varphi\dot{\varphi}) - \xi(2\dot{H} + 3H^2)\varphi^2 + p_\Lambda], \tag{31}$$

where H is the Hubble parameter, and ρ_Λ, p_Λ are the energy density and pressure of the holographic dark energy. The equation of motion of the scalar field is the modified Klein-Gordon equation

$$\ddot{\varphi} + 3H\dot{\varphi} + 6\xi(\dot{H} + 2H^2)\varphi = 0. \tag{32}$$

Replacing the holographic density (3) in (3), leads to the following Friedmann equation

$$H^2(1-\alpha) = \frac{\kappa^2}{3}(\frac{1}{2}\dot{\varphi}^2 + 6\xi H\varphi\dot{\varphi} + 3\xi H^2\varphi^2) + \beta\dot{H}. \tag{33}$$

Although the above equations may include the potential, in this case we show that the effect of accelerated expansion can be obtained without the need to introduce a potential. Due to the non-minimal coupling from (30), the effective gravitational coupling can be expressed as

$$G_{eff} = \frac{G}{1 - 8\pi G\xi\varphi^2}, \tag{34}$$

where G is the constant Newtonian coupling. The relative time variation of the gravitational coupling, obtained from (34) is given by

$$\frac{\dot{G}_{eff}}{G_{eff}} = -\frac{16\pi G\xi\varphi\dot{\varphi}}{1 - 8\pi G\xi\varphi^2}. \tag{35}$$

An appropriate solution of the equations (32) and (33) should give the relative variation of the gravitational coupling consistent with current observational bounds.

The equation (33) simplifies under the restriction $\alpha = 1$, and has an exact solution for the scalar field for $H = H_0 =$ const., corresponding to de Sitter expansion with $w_{eff} = -1$. Setting $\alpha = 1$ and $H = H_0$ the equations (32) and (33) become

$$\frac{1}{2}\dot{\varphi}^2 + 6\xi H_0\varphi\dot{\varphi} + 3\xi {H_0}^2\varphi^2 = 0, \tag{36}$$

$$\ddot{\varphi} + 3H_0\dot{\varphi} + 12\xi {H_0}^2\varphi = 0. \tag{37}$$

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If one assumes the scalar field of the form

$$\varphi = \varphi_0 e^{\eta H_0 t}, \quad (38)$$

then, the Eqs. (36) and (37) have two solutions corresponding to

$$\xi = \frac{1}{6}, \ \eta = -1 \ \text{ and } \ \xi = \frac{3}{16}, \ \eta = -\frac{3}{2} \quad (39)$$

Of special interest is the solution with $\xi = 1/6$ corresponding to the conformal coupling of the scalar field. This de Sitter solution is not possible in absence of the holographic energy density. So, the non-minimally coupled scalar field in the framework of the holographic energy density can produce the same effect as the cosmological constant, leading to a de Sitter expansion when the coupling constant takes the conformal value $\xi = 1/6$ and the scalar field evolves as $\varphi = \varphi_0 e^{\eta H_0 t}$.

For the solution (38), and using Eq. (35) we find the current value of $\frac{\dot{G}_{eff}}{G_{eff}}$ as

$$\frac{\dot{G}_{eff}}{G_{eff}} = -\frac{2\xi\, H_0 (\varphi_0/M_p)^2 e^{2\eta H_0 t}}{1 - \xi(\varphi_0/M_p)^2 e^{2\eta H_0 t}}. \quad (40)$$

Here we used $8\pi G = M_p^{-2}$ (M_p is the Planck mass) and H_0 is the current value of the Hubble parameter. For the case of the conformal coupling ($\xi = \frac{1}{6}, \eta = -1$), if we take $\varphi_0/M_p \simeq 10^{-3}$ and $t \sim H_0^{-1}$, then the relative variation of $\frac{\dot{G}_{eff}}{G_{eff}} \sim 10^{-18}\ yr^{-1}$, the gravitational coupling is of the order of , clearly satisfying the observational bounds.

Discussion

In the present work we considered the usual canonical minimally coupled scalar field with an additional source, given by the vacuum energy, modeled by the holographic density (**Granda and Oliveros,** 2008). It was found that, under exponential potential, in the vacuum background generated by the holographic principle, the model admits the same solutions of the minimally coupled scalar field for the power-law expansion, with the advantage that in the present case additionally appear phantom solutions (leading to future Big Rip singularities) without resorting to ghost degrees of freedom. In presence of matter with constant equation of state w_m, the model presents a rich variety of critical points that give de Sitter attractors, stable quintessence solutions and saddle points. Thus for instance, the point **P1** contains a de Sitter solution dominated by the holographic component, with marginal stability due to the eigenvalues (0,-3). This point also contains matter dominated solution with $w_{eff} = w_m$ which is a saddle point as in the case of absence of the holographic component. The point **P4** contains a de Sitter attractor dominated by the holographic component, and also contains stable quintessence ($-1 < w_{eff} < 0$) solutions where all the energy components contribute.

The point **P6** contains attractor solutions with accelerated expansion where the different components can give contributions to the energy density, and also contains a scalar field dominated solution which is a stable node with accelerated expansion. The de Sitter solution for this critical point is dominated by the holographic component and is a marginally stable fixed point with eigenvalues (-3, 0). These results show that the phase space of the autonomous system is richer than in the case of minimally coupled scalar field, giving rise to more accelerated expansion scenarios.

In the case of non-minimally coupled scalar field, the holographic dark energy as

given by (3), leads to interesting late time cosmological scenario: the model behaves as the cosmological constant, giving an exact de Sitter solution with dynamically evolving scalar field and without potential in the conformal coupling ($\xi = 1/6$) regime.

It is worth doing a further analysis of the dynamical system for the scalar field with non-minimal coupling, in the framework of the holographic principle, to study its critical points and check if there are critical points with phantom behavior since, according to current observations, the equation of state of the dark energy could be below the cosmological constant divide. It would be also of interest to analyze the dynamical system of the scalar field, model (1)-(3), with a potential different from exponential, for instance with power-law potential.

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Conflicts of interest

The author declare that there are no conflicts of interest related to the contents of this article.

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