

New universal 3-3-1 models

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Abstract

We review in a systematic way how anomaly free $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ models without exotic electric charges are constructed, using as basis closed sets of fermions which includes each one the particles and antiparticles of the electrically charged fields. Our analysis reproduce not only the known models in the literature, but also shows the existence of two new family independent ones not considered so far. © 2018. Acad. Colomb. Cienc. Ex. Fis. Nat.

Key words: Gauge models; Extensions to the standard model; Gauge anomalies.

Nuevos modelos 3-3-1 universales

Resumen

Se revisa de manera sistemática como se construyen los modelos sin cargas eléctricas exóticas basados en el grupo gauge local $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$, usando como base conjuntos cerrados de fermiones los cuales incluyen las partículas y las antipartículas de los campos eléctricamente cargados. Nuestro análisis no solo reproduce los modelos presentes en la literatura, sino que muestra la existencia de dos modelos nuevos independientes de familias no estudiados a la fecha. © 2018. Acad. Colomb. Cienc. Ex. Fis. Nat.

Palabras clave: Modelos gauge; Extensiones al modelo estándar; Anomalías gauge.

Introduction

The impressive success of the Standard Model (SM) based on the local gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ with the color sector $SU(3)_c$ confined and the flavor sector $SU(2)_L \otimes U(1)_Y$ hidden and broken spontaneously by the minimal Higgs mechanism (Donoghue, Golowich & Holstein, 2014), has not been able enough to provide, among others, explanation for several fundamental issues, among them the masses and mixing angles for both, the quark and the lepton sectors, and the abundance of dark matter and dark energy in the universe. Because of this, many physicists believe that it does not stand for the final theory, representing only an effective model originated from a more fundamental one.

Minimal extensions of the SM arise either by adding new fields, or by enlarging the local gauge group (adding a right handed neutrino field constitute its simple extension, something that ameliorate, but not solve the two problems mentioned above).

Simple extensions of the local gauge group consider an electroweak sector with an extra abelian symmetry $SU(2)_L \otimes U(1)_W \otimes U(1)_Z$ (Ponce, 1987), or either the so called left right symmetric model $SU(2)_L \otimes SU(2)_R \otimes U(1)_{(B-L)}$ and also $SU(3)_L \otimes U(1)_X$, being the last the one we are going to consider next (Pisano & Pleitez, 1992; Ponce, *et al.*, 2001).

3-3-1 Models

In what follows we assume that the gauge group is $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ (3-3-1 for short) in which the electroweak sector of the standard model $SU(2)_L \otimes U(1)_Y$ is extended to $SU(3)_L \otimes U(1)_X$. We also assume that, as in the SM, the color group $SU(3)_c$ is vector-like (free of anomalies) and that the left-handed quarks (color triplets) and left-handed leptons (color singlets) transform under the two fundamental representations of $SU(3)_L$ (the 3 and 3*).

Two classes of models will show up: universal models where the anomalies cancel in each family as in the SM, and family models where the anomalies cancel by an interplay between the several families.

For the 3-3-1 models, the most general electric charge operator in the extended electroweak sector is

$$Q = a\lambda_3 + \frac{1}{\sqrt{3}}b\lambda_8 + XI_3, \quad (1)$$

where λ_α , $\alpha = 1, 2, \dots, 8$ are the Gell-Mann matrices for $SU(3)_L$ normalized as $\text{Tr}(\lambda_\alpha \lambda_\beta) = 2\delta_{\alpha\beta}$ and $I_3 = \text{Dg}(1, 1, 1)$ is

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the diagonal 3×3 unit matrix. $a = 1/2$ if one assumes that the isospin $SU(2)_L$ of the SM is entirely embedded in $SU(3)_L$; b is a free parameter which fixes the model and the X values are obtained by anomaly cancellation. $X = 0$ for the 8 gauge fields A_μ^α of $SU(3)_L$, and thus they may be written as (Ponce, et al., 2001):

$$\sum_\alpha \lambda_\alpha A_\mu^\alpha = \sqrt{2} \begin{pmatrix} D_{1\mu}^0 & W_\mu^+ & K_\mu^{(b+1/2)} \\ W_\mu^- & D_{2\mu}^0 & K_\mu^{(b-1/2)} \\ K_\mu^{-(b+1/2)} & K_\mu^{-(b-1/2)} & D_{3\mu}^0 \end{pmatrix},$$

where $D_{1\mu}^0 = A_{\mu}^3/\sqrt{2} + A_{\mu}^8/\sqrt{6}$, $D_{2\mu}^0 = -A_{\mu}^3/\sqrt{2} + A_{\mu}^8/\sqrt{6}$, and $D_{3\mu}^0 = -2A_{\mu}^8/\sqrt{6}$. The upper indices on the gauge bosons stand for the electric charge of the particles, some of them being functions of the b parameter.

The Minimal Model

Pisano & Pleitez (1992) shown that, for $b = 3/2$, the following fermion structure is free of all the gauge anomalies: $\psi_{lL}^T = (v_l^0, l^-, l^+) \sim (1, 3, 0)$, $Q_{lL}^T = (d_l, u_l, X_l) \sim (3, 3^*, -1/3)$, $Q_{3L}^T = (u_3, d_3, Y) \sim (3, 3, 2/3)$, where $l = e, \mu, \tau$ is a family lepton index, $i = 1, 2$ for the first two quark families, and the numbers after the similarity sign means 3-3-1 representations. The right handed fields are $u_{aL}^c \sim (3^*, 1, -2/3)$, $d_{aL}^c \sim (3^*, 1, 1/3)$, $X_{iL}^c \sim (3^*, 1, 4/3)$ and $Y_L^c \sim (3^*, 1, -5/3)$, where $a = 1, 2, 3$ is the quark family index and there are two exotic quarks with electric charge $-4/3$ (X_i) and other with electric charge $5/3$ (Y).

This version is called minimal in the literature, because it does not make use of exotic leptons including possible right-handed neutrinos.

3-3-1 Models Without Exotic Electric Charges

If one wishes to avoid exotic electric charges in the fermion and boson sectors as the ones present in the minimal model, one must choose $b = 1/2$, in Eq. (1) as shown in Ponce, et al. (2001).

To start with, let us consider the following six closed sets of fermions, closed in the sense that they contain the antiparticles of all the charged particles

- $S_1 = [(v_e^0, e^-, E^-) \otimes e^+ \otimes E^+]_L$ with quantum numbers $(1, 3, -2/3)$; $(1, 1, 1)$ and $(1, 1, 1)$ respectively.
- $S_2 = [(e^-, v_e^0, N^0) \otimes e^+]_L$ with quantum numbers $(1, 3^*, -1/3)$ and $(1, 1, 1)$ respectively.
- $S_3 = [(d, u, U) \otimes u^c \otimes d^c \otimes U^c]_L$ with quantum numbers $(3, 3^*, 1/3)$; $(3^*, 1, -2/3)$; $(3^*, 1, 1/3)$ and $(3^*, 1, -2/3)$ respectively.
- $S_4 = [(u, d, D) \otimes u^c \otimes d^c \otimes D^c]_L$ with quantum numbers $(3, 3, 0)$; $(3^*, 1, -2/3)$; $(3^*, 1, 1/3)$ and $(3^*, 1, 1/3)$ respectively.
- $S_5 = [(N_1^0, E_1^+, e^+) \otimes E_1^- \otimes e^-]_L$ with quantum numbers $(1, 3^*, 2/3)$ $(1, 1, -1)$, and $(1, 1, -1)$ respectively.
- $S_6 = [(E_2^+, N_2^0, N_3^0) \otimes E_2^-]_L$ with quantum numbers $(1, 3, 1/3)$ and $(1, 1, -1)$ respectively.

Table 1. Anomalies for 3-3-1 fermion fields structures

Anomalies	S_1	S_2	S_3	S_4	S_5	S_6
$[SU(3)_C]^2 U(1)_X$	0	0	0	0	0	0
$[SU(3)_L]^2 U(1)_X$	-2/3	-1/3	1	0	2/3	1/3
$[Grav]^2 U(1)_X$	0	0	0	0	0	0
$[U(1)_X]^3$	10/9	8/9	-12/9	-6/9	-10/9	-8/9
$[SU(3)_L]^3$	1	-1	-3	3	-1	1

In the former sets N^0 and N_3^0 can play the role of the right handed neutrino ν_e^{0c} in an $SO(10)$ basis.

Notice that our approach here is different to the one presented in Ponce, et al. (2001), the difference being that only one $SU(3)_L$ triplet is used in each set, instead of the composite ones present in the former reference.

The several gauge anomalies calculated for these six sets are proportional to the values quoted in Table 1 (a common constant value or a Kronecker delta can be present in each row in the table). Notice that the anomaly values for S_1, S_2, S_3 and S_4 coincide with the ones presented in Ponce, et al. (2001), being the values for S_5 and S_6 new results.

Now, if we want to consider only one family of quarks, either the sets S_3 or S_4 are enough, but for 3 quark families, one of the following combinations must be used: $3S_3, 3S_4, (2S_3 + S_4)$ and $(S_3 + 2S_4)$.

Taking this into account and using the values in Table 1, we can see that with the particle sets under consideration it is impossible to construct one family (universal) models.

An unrealistic two family model is for example $S_1 + S_2 + S_3 + S_4$ [unrealistic because there is strong evidence for at least three families in nature (Donoghue, Golowich & Holstein, 2014)].

Right from Table 1 it is simple to read the following two three family models free of anomalies:

- Model A: $3S_2 + S_3 + 2S_4$.
- Model B: $3S_1 + 2S_3 + S_4$,

two well known models in the literature; A being named as a “Model with right-handed neutrinos” (Foot, et al., 1994; Benavides, et al., 2009; Montero, et al., 1993) and B named as a “Model with exotic charged leptons” (Özer, 1996; Ponce & Zapata, 2006; Salazar, et al., 2007).

The strategy now is to use the lepton sets S_1, S_2, S_5 and S_6 to build non vector-like new sets of leptons. Notice that Vector-like sets as for example $S_1 + S_5$ and $S_2 + S_6$ are free of anomalies and not suitable for constructing realistic models due to the non zero anomalies in the quark sector [Vector-like sets are free of anomalies by definition (Donoghue, Golowich & Holstein, 2014)].

The new lepton sets we want to consider are now:

- $S_7 = [(e^-, v_e^0, N_1^0) \otimes (N_2^0, E^+, e^+) \otimes E^-]_L$ with quantum numbers $(1, 3^*, -1/3)$; $(1, 3^*, 2/3)$ and $(1, 1, -1)$ respectively.

- $S_8 = [v_e^0, e^-, E^-] \otimes (E^+, N_1^0, N_2^0) \otimes e^+]_L$ with quantum numbers (1, 3, -2/3), (1, 3, 1/3) and (1, 1, 1) respectively.
- $S_9 = [(e^-, \nu_e, N_1^0) \otimes (E^-, N_2^0, N_3^0) \otimes (N_4^0, E^+, e^+)]_L$ with quantum numbers (1, 3*, -1/3); (1, 3*, -1/3) and (1, 3*, 2/3) respectively.
- $S_{10} = [\nu_e, e^-, E^-] \otimes (E_2^+, N_1^0, N_2^0) \otimes e^+ \otimes (N_3^0, E_2^-, E_3^-) \otimes E_1^+ \otimes E_3^+]_L$ with quantum numbers (1, 3, -2/3); (1, 3, 1/3); (111); (1, 3, -2/3); (111); (111), and (111) respectively.

The anomalies for these new lepton sets are given in Table 2. Again, the $SU(2)_L$ singlets N_1^0 in S_7 and S_9 , N_2^0 in S_8 and S_{10} and N_3^0 in S_9 can play the role of the right handed neutrino field ν_e^{0c} .

Notice that again $S_7 + S_8$ is Vector-like and because of this free of anomalies.

With the anomaly values in Table 1 and Table 2 new one family and three family models can be constructed. As a matter of fact, the following three family models show up:

- Model C: with unique lepton generation one $S_1 + S_2 + S_3 + 2S_4 + S_9$
- Model D: with unique lepton generation two $S_1 + S_2 + 2S_3 + S_4 + S_{10}$
- Model E: hybrid one $S_3 + 2S_4 + 2S_9 + S_{10}$
- Model F: hybrid two $2S_3 + S_4 + S_9 + 2S_{10}$

The phenomenology for the lepton sector of the two models with unique lepton generation (models C and D) has been presented by **Anderson & Sher** (2005). The phenomenology of the two hybrid models E and F have not been presented in the literature as far as we know.

Also, the following four one family models (universal) are readily constructed:

- Model G: carbon copy one $S_4 + S_9$
- Model H: carbon copy two $S_3 + S_{10}$
- Model I: carbon copy three $2S_2 + S_4 + S_5$
- Model J: carbon copy four $2S_1 + S_3 + S_6$

According to **Sanchez, et al.** (2001), the fermion structure of model G can be embedded in the unification group E(6), with some phenomenology of this structure already presented in the same reference.

In a similar way, the fermion structure of model H can be embedded in the gauge group $SU(6) \otimes U(1)$ as presented by **Martinez, et al.** (2001).

As can be seen, the first eight models A.....H have been already reported in the literature (**Ponce, et al.**, 2001); but models I and J are new ones!

The name ‘‘Carbon Copy’’ we adopted is due to the fact that realistic, 3 family models, can be constructed just by repeating three times the corresponding fermion structures, just as in the SM.

All the models presented here can also be enlarged by adding to each one of them an extra vector like structure ($S_1 + S_5$ and/or $S_2 + S_6$).

The fermion structures studied in this section are so rich, that they can be used to construct non universal models

Table 2. Anomalies for the 3-3-1 non Vector-Like lepton fields structures

Anomalies	S_7	S_8	S_9	S_{10}
$[SU(3)_C]^2 U(1)_X$	0	0	0	0
$[SU(3)_L]^2 U(1)_X$	1/3	-1/3	0	-1
$[Grav]^2 U(1)_X$	0	0	0	0
$[U(1)_X]^3$	-2/9	2/9	6/9	12/9
$[SU(3)_L]^3$	-2	2	-3	3

for several families (two, three, four, five, etc.). In particular, four family models using these structures were presented for the first time by **Ponce, et al.** (2010).

To conclude this section let us mention that the extension of this analysis to non universal three family models (work in progress) can be used to explain the experimental anomalies reported recently in the Large Hadron Collider.

The 3-3-1 model with righthanded neutrinos

To make explicit our notation, let us present here the Fermion structure of the most popular of the models analyzed here, the so called ‘‘model with right-handed neutrinos’’, introduced in the literature for the first time by **Montero, et al.** (1993) and **Foot, et al.** (1994). This model has the following 3-3-1 anomaly free fermion structure:

$$\begin{aligned} \psi_{lL}^T &= (l^-, \nu_l^0, N_l^0)_L \sim (1, 3^*, -1/3), \quad l_L^+ \sim (1, 1, 1), \\ Q_{iL}^T &= (u_i, d_i, D_i)_L \sim (3, 3, 0), \\ Q_{3L}^T &= (d_3, u_3, U)_L \sim (3, 3^*, 1/3), \end{aligned}$$

where $l = e, \mu, \tau$ is a family lepton index, N_{lL}^0 stands for electrically neutral Weyl states, and $i = 1, 2$ for the first two quark families. The right handed quark fields are

$$\begin{aligned} u_{aL}^c &\sim (3^*, 1, -2/3), \quad d_{aL}^c \sim (3^*, 1, 1/3), \\ D_{iL}^c &\sim (3^*, 1, 1/3), \quad U_L^c \sim (3^*, 1, -2/3), \end{aligned}$$

where again $a = 1, 2, 3$ is the quark family index. Once more, there are two exotic quarks with electric charge -1/3 (D_i) and other with electric charge 2/3 (U).

The extended phenomenology studies done for this model shows that N_l^0 , $l = e, \mu, \tau$ can be identified with the right-handed neutrinos ν_l^{0c} , which belong to the same 3-3-1 representation of the neutrino fields; so, lepton number is not a good quantum number for this particular model and it is not conserved (**Chang & Long**, 2006).

New universal 3-3-1 models

Let us see the fermion structure of these two new models which appears from our systematic analysis:

Model I. The particle content for this model is the following:

$$\begin{aligned} &[(e^-, \nu_e^0, N^0) \oplus e^+ \oplus (E_1^-, N_1^0, N_2^0) \oplus E_1^+]_L \oplus \\ &[(N_3^0, E_2^+, E_3^+) \oplus E_2^- \oplus E_3^-]_L \oplus \\ &[(u, d, D) \oplus u^c \oplus d^c \oplus D^c]_L \end{aligned}$$

A model with an extra down quark, three exotic electrons and five neutral Weyl states in each family.

Model J. The particle content for this model is the following

$$[(\nu_e^0, e^-, E_1^-) \oplus e^+ \oplus E_1^+ \oplus (N_1^0, E_2^-, E_3^-)]_L \oplus \\ [E_2^+ \oplus E_3^+ \oplus (E_4^+, N_2^0, N_3^0) \oplus E_4^-]_L \oplus \\ [(d, u, U) \oplus u^c \oplus d^c \oplus U^c]_L$$

A model with an extra up quark, four exotic electrons and four neutral Weyl states in each family.

The phenomenology for these two new models is being carried through right now by our research group and will be presented elsewhere.

Summary

In this paper we have done a systematic analysis in order to construct 3-3-1 models without exotic electric charges; ten models appear, eight of them already considered in our previous analysis (Ponce, *et al.*, 2001) and two new ones which we believe deserve special attention.

The next question is how to break the 3-3-1 gauge symmetry and provide with masses to the fermion fields in each model. The answer for this is a matter not settled yet. In Ponce, *et al.* (2003) the minimal set of Higgs scalars able to break the symmetry is presented, something called in the literature as the economical 3-3-1 model; unfortunately, it is not clear yet if this minimal set provides with masses to all the fermion fields in each model, something which is very unlike (Montero & Sánchez-Vega, 2015). A more realistic set of Higgs scalars able to break the symmetry and to provide masses to the fermion fields has been studied in detail (Ponce & Giraldo, 2001; Ponce, *et al.*, 2009).

A very attractive fact about the economical 3-3-1 structure is that in some versions of it, the troublesome Majoron is gauged away and is not present in the particle spectrum (Benavides, *et al.*, 2015).

Finally, let us mention that the phenomenology for the two new models presented is under consideration right now, and the results will be presented elsewhere.

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