Revista de la Academia Colombiana de Ciencias Exactas, Físicas y Naturales. 2026
doi: https://doi.org/10.18257/raccefyn.4111

Physical Sciences

Original article

Radiative neutrino masses in an abelian extension of the Standard Model

Masas radiativas de neutrinos en una extensión abeliana del Modelo Estándar

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Abstract

The seesaw mechanism is the preferred method to obtain light neutrino masses by introducing a Majorana mass matrix. Moreover, radiative corrections can be made to improve the predictions of the model. However, when such a Majorana mass matrix has no inverse, radiative corrections are the only mechanism to get massive neutrinos. Here, we show a non-universal abelian extension U(1)X of the Standard Model in which neutrinos acquire masses completely by radiative corrections through one-loop self-energies involving heavier right-handed neutrinos. Besides, the right-handed neutrino $v2R$ also gets mass through radiative corrections involving Majorana fermions. Finally, some limit cases are formulated depending on the hierarchy among vacuum expectation values and Majorana masses.

Keywords: Flavor problem; neutrino physics; extended scalar sectors; beyond Standard Model; fermion masses; inverse seesaw mechanism.

Resumen

El mecanismo de balancín es el método preferido para obtener masas de neutrinos ligeros mediante la introducción de una matriz de masas de Majorana. También pueden hacerse correcciones radiativas para mejorar las predicciones del modelo. Sin embargo, cuando la matriz de masas de Majorana no tiene inversa, las correcciones radiativas resultan ser el único mecanismo para obtener neutrinos masivos. Aquí se muestra una extensión abeliana no universal U(1)X del Modelo Estándar en la cual los neutrinos adquieren masa completamente mediante correcciones radiativas a través de autoenergías de un lazo que involucran neutrinos derechos más pesados. Además, el neutrino derecho $v2R$ también obtiene masa mediante correcciones radiativas que involucran fermiones de Majorana. Por último, se esbozan algunos casos límite dependiendo de la jerarquía entre los valores esperados del vacío y las masas de Majorana.

Palabras clave: Problema de sabor; física de neutrinos; sectores escalares extendidos; más allá del Modelo Estándar; masas de fermiones; mecanismo de balancín inverso.

Introduction

Since the very beginning of the search on particle physics, neutrinos have shown really special behaviors in comparison with other fundamental particles such as electrons, muons or quarks. Neutrino oscillations points one of their most important features, which is that they have really small but finite masses with only a left-handed

Citation: Mantilla-Serrano SF, et al. Radiative neutrino masses in an abelian extension of the Standard Model. Revista de la Academia Colombiana de Ciencias Exactas, Físicas y Naturales. 2026 Sep 21. doi: https://doi.org/10.18257/raccefyn.4111

Editor: Jairo Roa Rojas

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Received: May 27, 2026
Accepted: July 2, 2026
Published on line: September 21, 2026

Revista de la Academia Colombiana de Ciencias Exactas, Físicas y Naturales. 2026
doi: https://doi.org/10.18257/raccefyn.4111

chirality **Esteban *et al.***, 2017; "NuFIT 3.0", 2016, despite the Standard Model (SM)**Glashow**, 1961; **Salam**, 1968; **Weinberg**, 1967 demands the existence of both left and right to generate masses through the electroweak spontaneous symmetry breaking (SSB).

In this way, the most preferred scheme to obtain small neutrino masses is the well-known *seesaw mechanism* (SSM)**Aristizabal Sierra** and **Yaguna**, 2011; **Das**, **Okada**, and **Raut**, 2017a, 2017b; **Gell-Mann** and **Ramond**, 1979; **Grimus** and **Lavoura**, 2002; **Minkowski**, 1977; **Mohapatra** and **Senjanović**, 1980; **Schechter** and **Valle**, 1980; **Yanagida**, 1980 which proposes the existence of right-handed neutrinos as Majorana fermions. Since they are SM singlets, right-handed neutrinos are able to present Majorana masses whose scale is fixed in order to get the suited mass scales of neutrino masses according to neutrino oscillation data. However, the simple SSM requires Majorana masses for the right-handed neutrinos at 10^{14} GeV, near to the usual grand unification scale and evidently out of any experimental probe available today. In order to solve this unpleasant problem, the *inverse seesaw mechanism* (ISS) introduces a two-folded spectrum of right-handed neutrinos and consequently the Majorana mass scale can be lower by the addition of an additional scale at which right-handed neutrinos acquires mass**Bhupal Dev** and **Pilaftsis**, 2012; **Cárcamo Hernández**, **Cataño Mur**, and **Martinez**, 2014; **Cárcamo Hernández** and **Martinez**, 2016; **Cárcamo Hernández**, **Martinez**, and **Nisperuza**, 2015; **Carcamo Hernandez**, **Martinez**, and **Ochoa**, 2013; **Cataño**, **Martinez**, and **Ochoa**, 2012; **Das *et al.***, 2017; **Dias *et al.***, 2012; **Mohapatra**, 1986; **Mohapatra** and **Valle**, 1986.

One realization of the ISS is presented in Ref. **Mantilla** and **Martinez**, 2017. The particle content includes the three lepton doublets of the SM ℓ_L^α ($\alpha = e, \mu, \tau$), three right-handed Dirac neutrinos ν_R^α and three right-handed Majorana fermions $\mathcal{N}_R^\alpha$ (see Tab. 1). After the SSB the neutral sector gets the mass matrix $\mathbb{M}_N$ in the flavor basis $\mathbf{N}_L = (\nu_L^\alpha, \nu_R^{\alpha\,C}, \mathcal{N}_R^{\alpha\,C})$

$$\mathbb{M}_N = \begin{pmatrix} 0 & \boldsymbol{\mathcal{M}}_\nu^{\mathrm{T}} \\ \boldsymbol{\mathcal{M}}_\nu & \boldsymbol{\mathcal{M}}_{\mathcal{N}} \end{pmatrix} = \begin{pmatrix} 0 & \mathcal{M}_\nu^{\mathrm{T}} & 0 \\ \mathcal{M}_\nu & 0 & \mathcal{M}_{\mathcal{N}}^{\mathrm{T}} \\ 0 & \mathcal{M}_{\mathcal{N}} & M_{\mathcal{N}} \end{pmatrix}, \tag{1}$$

Performing the inverse SSM together with the assumptions $\mathcal{M}_{\mathcal{N}} \gg \mathcal{M}_\nu \gg M_{\mathcal{N}}$, we find that $\mathbb{M}_N$ can be diagonalized by blocks in the following way

$$\left(\mathbb{V}_{L,\mathrm{SS}}^N\right)^\dagger \mathbb{M}_N \mathbb{V}_{L,\mathrm{SS}}^N = \begin{pmatrix} m_\nu & 0 & 0 \\ 0 & m_N & 0 \\ 0 & 0 & m_{\tilde{N}} \end{pmatrix} \tag{2}$$

spanned in the mass basis $\mathbf{n}_L = (\nu_L^i, N_L^i, \tilde{N}_L^i)$ $(i = 1, 2, 3)$, where the resultant 3×3 blocks are**Cataño**, **Martinez**, and **Ochoa**, 2012; **Dias *et al.***, 2012

$$\begin{aligned} m_\nu &= \mathcal{M}_\nu^{\mathrm{T}} \left(\mathcal{M}_{\mathcal{N}}\right)^{-1} M_{\mathcal{N}} \left(\mathcal{M}_{\mathcal{N}}^{\mathrm{T}}\right)^{-1} \mathcal{M}_\nu, \\ M_N &\approx \mathcal{M}_{\mathcal{N}} - M_{\mathcal{N}}, \quad M_{\tilde{N}} \approx \mathcal{M}_{\mathcal{N}} + M_{\mathcal{N}}. \end{aligned} \tag{3}$$

The left-handed or active neutrinos ν_L^α received tiny contributions from the right-handed or sterile species directly proportional to $M_{\mathcal{N}}$ and inversely to $\mathcal{M}_{\mathcal{N}}$, so as they get into the light neutrinos ν_L^i at thousandths of eV. On the contrary, the sterile species ν_R^α and $\mathcal{N}_R^\alpha$ mixed them together into the heavy neutrinos N_R^i and $\tilde{N}_R^i$ at units of TeV quasi-degenerated in M_N.

Regarding one-loop corrections, the Ref. **Aristizabal Sierra** and **Yaguna**, 2011 shows that neutrino mass matrix can be expressed by

$$\mathbb{M}_N = \mathbb{M}_N^{\mathrm{tree}} + \mathbb{M}_N^{1-\mathrm{loop}}, \tag{4}$$

where $\mathbb{M}_N^{1-\text{loop}}$ contains mass terms obtained from one-loop corrections

$$\mathbb{M}_N^{1-\text{loop}} = \begin{pmatrix} \delta\boldsymbol{\mathcal{M}}_L & \delta\boldsymbol{\mathcal{M}}_D^{\mathrm{T}} \\ \delta\boldsymbol{\mathcal{M}}_D & \delta\boldsymbol{\mathcal{M}}_R \end{pmatrix}. \tag{5}$$

In this way, active neutrinos acquire masses through seesaw contributions at tree-level as well as through one-loop corrections

$$m_\nu = -\boldsymbol{\mathcal{M}}_\nu^{\mathrm{T}} \boldsymbol{\mathcal{M}}_{\mathcal{N}}^{-1} \boldsymbol{\mathcal{M}}_\nu + \delta\boldsymbol{\mathcal{M}}_L. \tag{6}$$

The one-loop mass $\delta\boldsymbol{\mathcal{M}}_L$ is obtained from radiative corrections on self-energy of left-handed neutrinos $\Sigma_L(p^2 = 0)$. It has contributions from Z_μ, G_Z and h,

$$-i\Sigma(0) = -i\Sigma^Z(0) - i\Sigma^{Gz}(0) - i\Sigma^h(0), \tag{7}$$

and after regularizing the integrals involved in loop diagrams the correction turns out to be

$$\begin{aligned} \delta\boldsymbol{\mathcal{M}}_L = {} & \boldsymbol{\mathcal{M}}_\nu^{\mathrm{T}} \boldsymbol{\mathcal{M}}_{\mathcal{N}}^{-1} \\ & \left\{ \frac{g^2}{64\pi^2 M_W^2} \left[m_h^2 \ln\left(\frac{\boldsymbol{\mathcal{M}}_{\mathcal{N}}^2}{m_h^2}\right) + 3m_Z^2 \ln\left(\frac{\boldsymbol{\mathcal{M}}_{\mathcal{N}}^2}{m_Z^2}\right)\right]\right\} \boldsymbol{\mathcal{M}}_\nu, \end{aligned} \tag{8}$$

in which the heavy modes have been integrated out. Compared with tree-level expression, such a correction is suppressed by $\frac{1}{16\pi^2} \ln\left(\frac{\boldsymbol{\mathcal{M}}_{\mathcal{N}}^2}{m_Z^2}\right)$, so as it could contribute to the actual mass of light neutrinos.

The model presented in this article has received some modification to forbid seesaw mechanisms at tree-level because one of the sterile neutrinos does not get mass and the mass matrix $\boldsymbol{\mathcal{M}}_{\mathcal{N}}$ does not have inverse. Consequently, it is mandatory to obtain masses through radiative corrections, in contrast with Ref. **Aristizabal Sierra** and **Yaguna**, 2011 for normal SSM and **Bhupal Dev** and **Pilaftsis**, 2012 for ISS, where seesaw is allowed at tree level and one-loop corrections are done in order to improve the predictability of the models. The section introduces the scalar and leptonic content employed in the article as well as the neutral lepton Yukawa Lagrangian. Then, section presents one-loop radiative corrections to get Majorana masses for light ν_L^α and heavy ν_R^α species. Finally, some conclusions are outlined in section .

doi: https://doi.org/10.18257/raccefyn.4111

Scalar and leptonic content

Table 1. Bosonic and leptonic sector of the model with forbidden seesaw mechanism. The notation $X^{\pm}$ means the value of the X charge and the $\mathbb{Z}_2$ parity. The VEVs have the hierarchy $v_\chi \ll v_1 > v_2 > v_3$ **Mantilla** and **Martinez**, 2017.

Fermion	$X^{\pm}$	Bosons	$X^{\pm}$
Lepton doublets		Scalar doublets	
$\ell_L^e = \begin{pmatrix} \nu^e \\ e^e \end{pmatrix}_L$	0^+	$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{h_1+v_1+i\eta_1}{\sqrt{2}} \end{pmatrix}$	$+2/3^+$
$\ell_L^\mu = \begin{pmatrix} \nu^\mu \\ e^\mu \end{pmatrix}_L$	0^+	$\Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{h_2+v_2+i\eta_2}{\sqrt{2}} \end{pmatrix}$	$+1/3^-$
$\ell_L^\tau = \begin{pmatrix} \nu^\tau \\ e^\tau \end{pmatrix}_L$	-1^-	$\Phi_3 = \begin{pmatrix} \phi_3^+ \\ \frac{h_3+v_3+i\eta_3}{\sqrt{2}} \end{pmatrix}$	$+1/3^+$
Lepton singlets		Scalar singlet	
ν_R^e	$+1/3^+$	$\chi = \frac{\xi_\chi+v_\chi+i\zeta_\chi}{\sqrt{2}}$	$-1/3^+$
ν_R^μ	$+1/3^+$	σ	$-1/3^-$
$\mathcal{N}_R^\tau$	$+1/3^+$		
Majorana fermions		Gauge bosons	
$\mathcal{N}_R^e$	0^+	$W_\mu^{\pm}$, W_μ^0	0^+
ν_R^τ	0^+	B_μ, Z'_μ	

The Ref. **Mantilla** and **Martinez**, 2017 introduces a new nonuniversal abelian extension to the SM where the fermion mass hierarchy is addressed. The majority of fermions acquire masses without radiative corrections, even light and heavy neutrinos through SSMs allowed by the Yukawa couplings of the model. However, the model presented here differs from the previous one in the neutral sector. The neutrinos $\nu_R^\tau \leftrightarrow N_R^3$ were interchanged so as the neutral lepton Yukawa Lagrangian turns out to be

$$\begin{aligned} -\mathcal{L}_N = {} & h_{3\nu}^{ee}\overline{\ell_L^e}\tilde{\Phi}_3\nu_R^e + h_{3\nu}^{e\mu}\overline{\ell_L^e}\tilde{\Phi}_3\nu_R^\mu + h_{3N}^{e\tau}\overline{\ell_L^e}\tilde{\Phi}_3\mathcal{N}_R^\tau \\ & + h_{3\nu}^{\mu e}\overline{\ell_L^\mu}\tilde{\Phi}_3\nu_R^e + h_{3\nu}^{\mu\mu}\overline{\ell_L^\mu}\tilde{\Phi}_3\nu_R^\mu + h_{3N}^{\mu\tau}\overline{\ell_L^\mu}\tilde{\Phi}_3\mathcal{N}_R^\tau \\ & + g_{\chi N}^{ee}\overline{\nu_R^{eC}}\tilde{\chi}\mathcal{N}_R^e + g_{\chi N}^{\mu e}\overline{\nu_R^{\mu C}}\tilde{\chi}\mathcal{N}_R^e + g_{\chi N}^{\tau e}\overline{\mathcal{N}_R^{\tau C}}\tilde{\chi}\mathcal{N}_R^e \\ & + g_{\chi N}^{e\tau}\overline{\nu_R^{eC}}\tilde{\sigma}\nu_R^\tau + g_{\chi N}^{\mu\tau}\overline{\nu_R^{\mu C}}\tilde{\chi}\nu_R^\tau + g_{\chi N}^{\tau\tau}\overline{\mathcal{N}_R^{\tau C}}\tilde{\chi}\nu_R^\tau \\ & + \frac{M_N^{ee}}{2\sqrt{2}}\overline{\mathcal{N}_R^{eC}}\mathcal{N}_R^e + \frac{M_N^{e\tau}}{2\sqrt{2}}\overline{\mathcal{N}_R^{eC}}\nu_R^\tau + \frac{M_\nu^{\tau\tau}}{2\sqrt{2}}\overline{\nu_R^{\tau C}}\nu_R^\tau + \text{h.c.}, \end{aligned} \tag{9}$$

where $\widetilde{\Phi} = i\sigma_2\Phi^*$ are the scalar doublet conjugates and the Majorana mass is denoted by $M_{\mathcal{N}}^{ij}$. The neutral sector of the model involves Dirac and Majorana masses in its Yukawa Lagrangian. They can be described in the flavor and mass bases which are, respectively,

$$\begin{aligned} \mathbf{N}_L &= (\nu_L^{e,\mu,\tau}, \nu_R^{e,\mu,\tau\,C}, \mathcal{N}_R^{e,\tau\,C}), \\ \mathbf{n}_L &= (\nu_L^{1,2,3}, N_R^{1,2,3^C}, \mathcal{N}_R^{1,3\,C}). \end{aligned} \tag{10}$$

After the SSB the mass terms of the neutral sector in the flavor basis are

$$-\mathcal{L}_N = \frac{1}{2}\overline{\mathbf{N}_L^C}\mathbb{M}_N\mathbf{N}_L, \tag{11}$$

where the mass matrix has the following block structure

$$\mathbb{M}_N = \begin{pmatrix} 0 & \boldsymbol{\mathcal{M}}_\nu^{\mathrm{T}} \\ \boldsymbol{\mathcal{M}}_\nu & \boldsymbol{\mathcal{M}}_{\mathcal{N}} \end{pmatrix}, \tag{12}$$

doi: https://doi.org/10.18257/raccefyn.4111

where the block $\boldsymbol{\mathcal{M}}_\nu$ is

$$\mathcal{M}_\nu^{\mathrm{T}} = \frac{v_3}{\sqrt{2}} \left(\begin{array}{ccc|cc} h_{3\nu}^{ee} & h_{3\nu}^{e\mu} & 0 & 0 & h_{3N}^{e\tau} \\ h_{3\nu}^{\mu e} & h_{3\nu}^{\mu\mu} & 0 & 0 & h_{3N}^{\mu\tau} \\ 0 & 0 & 0 & 0 & 0 \end{array} \right), \tag{13}$$

while the block $\boldsymbol{\mathcal{M}}_\mathcal{N}$ turns out to be

$$\mathcal{M}_\mathcal{N} = \frac{v_\chi}{\sqrt{2}} \left(\begin{array}{ccc|cc} 0 & 0 & g_{\chi N}^{e\tau} & g_{\chi N}^{ee} & 0 \\ 0 & 0 & g_{\chi N}^{\mu\tau} & g_{\chi N}^{\mu e} & 0 \\ g_{\chi N}^{e\tau} & g_{\chi N}^{\mu\tau} & \tilde{M}_\nu^{\tau\tau} & \tilde{M}_N^{e\tau} & g_{\chi N}^{\tau\tau} \\ \hline g_{\chi N}^{ee} & g_{\chi N}^{\mu e} & \tilde{M}_N^{e\tau} & \tilde{M}_N^{ee} & g_{\chi N}^{\tau e} \\ 0 & 0 & g_{\chi N}^{\tau\tau} & g_{\chi N}^{\tau e} & 0 \end{array} \right) \tag{14}$$

where $\tilde{M}_N^{ij} = M_N^{ij}/v_\chi$. The determinant of $\mathcal{M}_\mathcal{N}$ is null. Moreover, the active neutrino ν_L^τ is disconnected from the rest of the neutral fermion system, leaving it massless and constraining the model to inverse ordering (IO)**Esteban *et al*.**, 2017; "NuFIT 3.0", 2016 with $\nu_L^\tau = \nu_L^3$. It is important to note that in the Yukawa Lagrangian (9) the leptonic doublet ℓ_L^τ does not appear due to its X-charge which was assigned in order to cancel chiral anomalies. Consequently, it cannot acquire mass neither by see-saw nor radiative corrections.

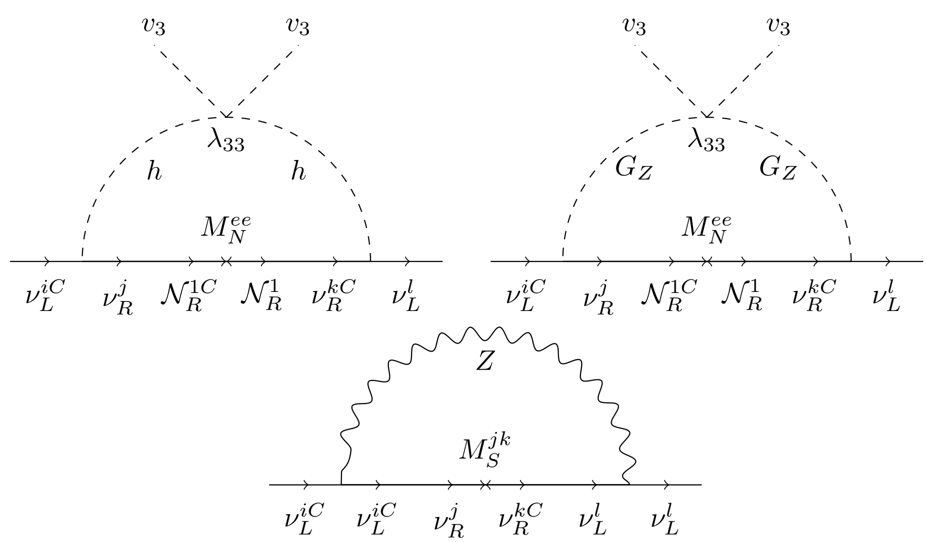

Figure 1. Radiative corrections at one-loop in the self-energy of ν_L where $i, l = e, \mu$ and $j, k = e, \mu, \tau$.

Radiative masses

The impossibility to perform the SSM in the neutral sector of the model implies massless active neutrinos at tree level. However, there exist one-loop diagrams which are suitable to get Majorana masses for active neutrinos involving right-handed neutrinos and the lepton number violation Majorana mass terms M_N. The next sections show how the model accounts for ν_L^α masses as well as right-handed neutrinos.

Revista de la Academia Colombiana de Ciencias Exactas, Físicas y Naturales. 2026
doi: https://doi.org/10.18257/raccefyn.4111

*Radiative mass for active neutrinos ν_L Since there is one right-handed neutrino massless at tree-level, the SSM is forbidden and the masses of light neutrinos have to be obtained through radiative corrections at one-loop level. The main contributions to the self-energy of ν_L^α come from SM fields: the gauge boson Z_μ, its Goldstone boson G_Z and the Higgs boson h, so as**Aristizabal Sierra** and **Yaguna**, 2011; **Bhupal Dev** and **Pilaftsis**, 2012; **Grimus** and **Lavoura**, 2002

$$-i\Sigma(p^2=0) = -i\Sigma^Z(0) - i\Sigma^{Gz}(0) - i\Sigma^h(0). \tag{15}$$

The contributions from non-SM fields are suppressed because of the hierarchy among v, the electroweak VEV, and v_χ at TeV scale. For instance, in the scalar sector the flavor basis $\mathbf{h} = (h_1, h_2, h_3, \xi_\chi)$ is connected with the mass basis $\mathbf{H} = (h, H_1, H_2, \mathcal{H})$ through $\mathbf{H} = R_{\text{even}}\mathbf{h}$ (the Higgs potential as well as scalar masses and mixings are explained in detail in Ref. **Mantilla** and **Martinez**, 2017). After the SSB, there are four CP-even scalar bosons: the lightest corresponding with the SM Higgs v, two intermediate scalars H_1 and H_2 at TeV scale and the heaviest $\mathcal{H}$ associated with the SSB of $\mathrm{U}(1)_X$. The mixing matrix R_{even} can be expressed as

$$R_{\text{even}} = \begin{pmatrix} 1 & s_{12}^h & s_{13}^h & s_{14}^h \\ -s_{12}^h & 1 & 0 & s_{24}^h \\ -s_{13}^h & 0 & 1 & s_{34}^h \\ -s_{14}^h & -s_{24}^h & -s_{34}^h & 1 \end{pmatrix} \tag{16}$$

where the mixing angles are given by

$$s_{i4}^h \approx \frac{v_i}{v_\chi}, \qquad s_{12}^h \approx \frac{v_2}{v_1}, \qquad s_{13}^h \approx \frac{v_3}{v_1}. \tag{17}$$

The most suppressed mixings involve s_{i4}^h because $v_i \ll v_\chi$, followed by s_{13}^h and s_{12}^h. Moreover, since light neutrinos couple with h_3 (look at the two first rows in Eq. (9)), which is expressed in the eigenstate basis as the superposition of h and the heavier scalars H_2 and $\mathcal{H}$, then

$$h_3 \approx H_2 + s_{13}^h h - s_{34}\mathcal{H}, \tag{18}$$

the less suppressed contribution to light neutrinos self-energy comes from the lightest scalar, h, with the mixing angle s_{13}^h.

Now, regarding the diagrams in Fig. 1, the fermionic propagator inside the loop is composed by four Yukawa coupling constants, two mass insertions between ν_R and $\mathcal{N}_R$ and the Majorana mass M_N^{ee} which violates lepton number conservation and connects ν_L with ν_L^C**Aristizabal Sierra** and **Yaguna**, 2011; **Bhupal Dev** and **Pilaftsis**, 2012. It is possible to associate an effective mass M_S to the fermionic propagator

$$M_S^{jk} = g_{\chi N}^{ke} g_{\chi N}^{je*} \frac{v_\chi^2}{2M_N^{ee}}. \tag{19}$$

The algebraic expression of the first diagram in Fig. 1, the self-energy involving the Higgs boson, is

$$\begin{aligned} P_R i[\Sigma(\not{p})^h]^{li} P_R = \lambda_{33} v_3^2 \frac{h_{3\nu}^{lk} s_{13}^h}{\sqrt{2}} P_R \\ \int \frac{dk}{(2\pi)^4} \frac{\not{p} - \not{k} + M_S^{kj}}{(p-k)^2 - (M_S^{kj})^2} \frac{1}{(k^2 - m_h^2)^2} P_R \frac{h_{3\nu}^{lk*} s_{13}^h}{\sqrt{2}} \end{aligned} \tag{20}$$

where λ_{33} is the quartic coupling constant of $(\Phi_3^\dagger \Phi_3)^2$ in the Higgs potential which can be approximated by

$$\lambda_{33} \approx \frac{m_h^2}{4M_W^2} \tag{21}$$

On the other hand, from the expressions of the second and third diagrams in Fig. 1, the self-energy involving G_Z and Z_μ are given by

$$
\begin{aligned}
&P_R i[\Sigma(\not{p})^Z]^{li} P_R = \frac{g^2}{4c_W^2}\frac{h_{3\nu}^{lk} v_3^h}{\sqrt{2}} P_R \\
&\gamma_\mu \int \frac{dk}{(2\pi)^4}\frac{\not{p}-\not{k}}{(p-k)^2}\frac{\not{p}-\not{k}+M_S^{kj}}{(p-k)^2-(M_S^{kj})^2} \\
&\frac{\not{p}-\not{k}}{(p-k)^2} P_R \frac{g^{\mu\nu}}{(k^2-m_Z^2)^2}\gamma_\nu \frac{h_{3\nu}^{lk*} v_3^h}{\sqrt{2}}.
\end{aligned} \tag{22}
$$

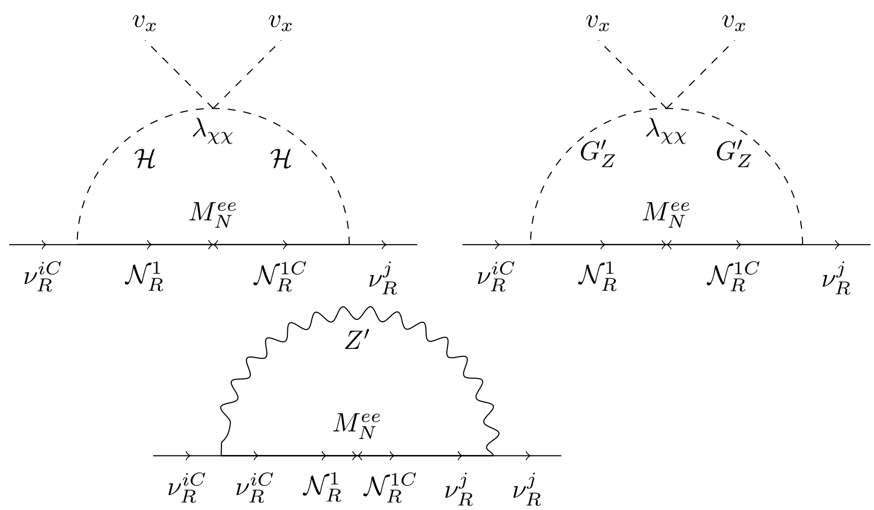

Figure 2. Radiative corrections at one-loop in the self-energy of ν_R where $i, j = e, \mu, \tau$.

After integrating both contributions, the resulting mass for light neutrinos is

$$
\begin{aligned}
M_\nu^{li} = &\frac{\alpha_W}{64\pi M_W^2}\left(h_{3\nu} g_{\chi N} g_{\chi N}^* h_{3\nu}^*\right)^{li} \frac{v_3^2 v_\chi^2}{M_N^{ee}}(s_{13}^h)^2 \\
&\left\{\frac{m_h^2}{M_S^2-m_h^2}\ln\frac{M_S^2}{m_h^2} + 3\frac{m_Z^2}{M_S^2-m_Z^2}\ln\frac{M_S^2}{m_Z^2}\right\}.
\end{aligned} \tag{23}
$$

The expression can be simplified by defining $\tilde{M}_S = v_\chi^2/M_N^{ee}$ and $s_\gamma = v_3/v$, and by replacing $M_W = gv/2$ so as

$$
\frac{\alpha_W v_3^2}{64\pi M_W^2} = \frac{1}{64\pi^2}\frac{v_3^2}{v_\chi^2} = \frac{1}{64\pi^2}\sin_\gamma^2 \tag{24}
$$

and the mass matrix is

$$
\begin{aligned}
M_\nu^{li} = &\frac{\sin_\gamma^2 (s_{13}^h)^2}{64\pi^2}\tilde{M}_S \left(h_{3\nu} g_{\chi N} g_{\chi N}^* h_{3\nu}^*\right)^{li} \\
&\left\{\frac{m_h^2}{M_S^2-m_h^2}\ln\frac{M_S^2}{m_h^2} + 3\frac{m_Z^2}{M_S^2-m_Z^2}\ln\frac{M_S^2}{m_Z^2}\right\}.
\end{aligned} \tag{25}
$$

Some interesting limits on M_ν^{li} can be drafted depending on the hierarchy between M_N^1 and v_χ. The fist one assumes $g_{\chi N} v_\chi \ll M_N^{ee}$, so the mass matrix turns out to

Revista de la Academia Colombiana de Ciencias Exactas, Físicas y Naturales. 2026
doi: https://doi.org/10.18257/raccefyn.4111

be

$$M_\nu^{li} = \frac{s_\gamma^2 (s_{13}^h)^2}{16\pi^2} \frac{\left(h_{3\nu} g_{\chi N} g_{\chi N}^* h_{3\nu}^*\right)^{li}}{g_{\chi N}^{ke} g_{\chi N}^{je*} g_{\chi N}^{je} g_{\chi N}^{ke*}} \left\{ \frac{m_h^2 + 3m_Z^2}{\tilde{M}_S} \ln \frac{\tilde{M}_S^2}{m_h^2} \right\} \tag{26}$$

where $\tilde{M}_S$ can be large as it is required to obtain light neutrinos. On the other hand, when $M_N^{ee} \ll g_{\chi N} v_\chi \ll m_h$

$$M_\nu^{li} = \frac{s_\gamma^2 (s_{13}^h)^2}{8\pi^2} \left(h_{3\nu} g_{\chi N} g_{\chi N}^* h_{3\nu}^*\right)^{li} \tilde{M}_S \ln \frac{m_h^2}{\tilde{M}_S^2} \tag{27}$$

where, in this case, $\tilde{M}_S$ gets small in consistency with light neutrinos.

Radiative mass for the massless ν_R

An analogous calculation is done on right-handed sector by doing some replacements on radiative correction and similar expressions are obtained, where the SM scalar and gauge fields h, G_Z and Z_μ have been replaced by the new $\mathcal{H}$, G'_Z and Z'_μ, respectively. The contributions from the SM Higgs boson h get suppressed because of the small mixing angles v_i/v_χ with $\mathcal{H}$, which receives major contributions from ξ_χ

$$\mathcal{H} = \xi_\chi - \sum_{i=1}^{3} \frac{v_i h_i}{v_\chi}. \tag{28}$$

Similarly, Z_μ does not contribute importantly to the self-energy of ν_R because of the mixing v^2/v_χ^2 with Z'_μ, so SM bosons can be neglected in first approximation.

Since the diagrams of Figs. 1 and 2 are similar under the replacements $(h, G_Z, Z_\mu) \to (\mathcal{H}, G'_Z, Z'_\mu)$ with the suited coupling constants, the resulting mass matrix for right-handed neutrinos is

$$M_R^{ki} = \frac{g_X}{64\pi^2 {M'_Z}^2} g_{\chi N}^{kj} g_{\chi N}^{ji*} \frac{v_\chi^2}{M_N^{ee}} \left\{ \frac{m_h^2}{M_S^2 - M_{\mathcal{H}}^2} \ln \frac{M_S^2}{M_{\mathcal{H}}^2} + 3 \frac{m_Z^2}{M_S^2 - M_{Z'}^2} \ln \frac{M_S^2}{M_{Z'}^2} \right\} \tag{29}$$

Similarly with the left-handed neutrino sector, some limits can be drafted. When $M_{\mathcal{H},Z'} \approx v_\chi \ll M_N^{ee}$

$$M_R^{ki} = \frac{3}{16\pi^2} g_{\chi N}^{kj} g_{\chi N}^{ji*} \frac{M_{Z'}^4}{(M_N^{ee})^3} \ln \left(\frac{(M_N^{ee})^2}{M_{Z'}^2} \right) \tag{30}$$

where the mass of the non-SM gauge boson Z'_μ, given by $M_{Z'} \approx g_X v_\chi/3$, has been replaced**Mantilla** and **Martinez**, 2017. This case yields the massless ν_R^2 as a light right-handed sterile neutrino. On the other hand, when $M_{\mathcal{H},Z'} \gg M_N^{ee}$

$$M_R^{ki} = \frac{3}{16\pi^2} g_{\chi N}^{kj} g_{\chi N}^{ji*} \frac{M_{Z'}^2}{(M_N^{ee})} \ln \left(\frac{M_{Z'}^2}{(M_N^{ee})^2} \right) \tag{31}$$

and the former massless ν_R^2 would get mass near to the TeV scale.

One interesting feature in one-loop mass expressions is the presence of terms $\frac{m_h^2}{(M_S)}$ or $\frac{M_{Z'}^2}{(M_N^{ee})}$, suggesting the existence of a seesaw-like mechanism inside self-energy loops. Actually, those factors determine the mass scale of neutrinos which have acquired masses through radiative corrections.

Discussion and Conclusions

The evidence of neutrino oscillations, as well as the upper limits on absolute neutrino masses have yielded stringent constraints in models which address neutrino physics. The most preferred framework to understand neutrino masses is the seesaw mechanism, whose main hypothesis is the existence of sterile right-handed neutrinos as Majorana fermions, which have lepton number violating masses at a higher scale so as left-handed neutrinos turns out to be Majorana fermions with masses at eV scale. Moreover, the introduction of a second set of right-handed neutrinos, together with the existence of a second VEV at TeV allows lower Majorana mass scale to accessible experiment energies. However, there exist the scenario in which the Majorana mass matrix does not have inverse matrix, yielding massless right-handed neutrinos as well as the impossibility to obtain neutrino masses at tree level. In this way, radiative corrections comprise the principal mechanism to get light massive neutrinos.

The model presented here implies such a scenario where radiative corrections are the main mechanism to get masses. There are two kinds of loops, the self-energies of ν_L involving the SM bosons Z_μ, G_Z and h together with right-handed neutrinos ν_R and $\mathcal{N}_R$, and the self-energy of ν_R^2 involving the non-SM bosons Z'_μ, G'_Z and $\mathcal{H}$ with the Majorana fermions $\mathcal{N}_R$. Furthermore, depending on the hierarchy between the VEV v_χ at TEV scale and the Majorana mass M_N^{ee} which would be at a higher or lower scale. Such limits were obtained in section and are outlined in Tab. 2.

Consequently, the model shows how a scenario in which Majorana mass matrix does not have inverse implies massless neutrinos at tree level and massive at one-loop level through radiative corrections involving heavy modes and bosons, in contrast with the majority of models where neutrinos get masses at tree level by seesaw mechanisms.

Table 2. Special cases given by the hierarchy between v_χ and M_N^{ee}. The left column resembles ISS while the right one the double SSM.

	$v_\chi \gg M_N^{ee}$	$v_\chi \ll M_N^{ee}$
ν_L	$\frac{s_\gamma^2 (s_{13}^h)^2}{16\pi^2} \frac{m_h^2}{\tilde{M}_S} \ln\left(\frac{\tilde{M}_S^2}{m_h^2}\right)$	$\frac{s_\gamma^2 (s_{13}^h)^2}{16\pi^2} \tilde{M}_S \ln\left(\frac{m_h^2}{\tilde{M}_S^2}\right)$
N_R^1	$v_\chi - M_N^{ee}$	$\frac{v_\chi^2}{M_N^{ee}}$
N_R^2	$\frac{3}{16\pi^2} \frac{M_{Z'}^2}{M_N^{ee}} \ln\left(\frac{M_N^{ee}}{M_{Z'}^2}\right)$	$\frac{3}{16\pi^2} \frac{M_{Z'}^4}{(M_N^{ee})^3} \ln\left(\frac{M_{Z'}^2}{M_N^{ee}}\right)$
N_R^3	$v_\chi - M_N^{\tau\tau}$	$\frac{v_\chi^2}{M_N^{\tau\tau}}$
$\mathcal{N}_R^1$	$v_\chi + M_N^{ee}$	$M_N^{ee} + \frac{v_\chi^2}{M_N^{ee}}$
$\mathcal{N}_R^3$	$v_\chi + M_N^{\tau\tau}$	$M_N^{\tau\tau} + \frac{v_\chi^2}{M_N^{\tau\tau}}$

Author contributions

S.F.M.: calculated the Lagrangians of the model and Feynman's rules. He wrote a draft of the paper. R.M.: calculated all radiation corrections and complete the English version. J.M.: reviewed the calculations and worked on the LaTeX versions.

Conflicts of interests:

The authors declare that we have no conflict of interest.

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