Información suplementaria

A winsorized adaptive rank test for location when sampling from asymmetric distributions

Una prueba de rangos adaptativa winsorizada para localización] en muestras de distribuciones asimétricas

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Contenido

Appendix. A. The Generalized Lambda Distribution (GLD)

The GLD is defined through its percentile function:

$$F^{-1}(y) = \lambda_1 + \frac{y^{\lambda_3} - (1-y)^{\lambda_4}}{\lambda_2}, \quad (A.1)$$

where λ_1, λ_2, λ_3, λ_4 are the location, scale, skewness and kurtosis parameters respectively. Its probability density function is given by:

$$f(x) = \frac{\lambda_2}{\lambda_3 y^{\lambda_3 - 1} + \lambda_4 (1-y)^{\lambda_4 - 1}}, \quad en \; x = F^{-1}(y).$$

The regions in which the λ_i, $i = 1, 2, 3, 4$, yield valid distributions and explicit expressions for the moments, so as many other properties of the distribution can be find in Karian and Dudewicz 2000.

Appendix B. Proofs. Conditional Mean and Variance of the Proposed Test Statistic under H0.

Proof. Valid for $g(p) = p$ or $g(p) = \sqrt{p}$.

$$E[\overline{BW}|p] = E\left[\frac{1}{N} \sum_{i=1}^{N} min\left\{\left(\frac{Ri}{N+1}\right)^{g(p)}, 1 - \gamma\right\} s(X_i) \mid p\right]$$

$$= \frac{1}{N} \sum_{i=1}^{N} min\left\{\left(\frac{Ri}{N+1}\right)^{g(p)}, 1 - \gamma\right\} E[s(X_i)]$$

$$= \frac{1}{2} \sum_{i=1}^{N} min\left\{\left(\frac{Ri}{N+1}\right)^{g(p)}, 1 - \gamma\right\} \frac{1}{N}$$

$$\xrightarrow{N \to \infty} \frac{1}{2} \int_0^1 min\{u^{g(p)}, 1 - \gamma\} \, du$$

$$= \frac{(1-\gamma)}{2} \left[1 - \left(\frac{g(p)}{g(p)+1}(1-\gamma)^{\frac{1}{g(p)}}\right)\right],$$

$$N \, Var[\overline{BW} \mid p] = NVar\left[\frac{1}{N}\sum_{i=1}^{N} min\left\{\left(\frac{Ri}{N+1}\right)^{g(p)}, 1-\gamma\right\}s(X_i) \mid p\right]$$

$$= \frac{N}{N^2}\sum_{i=1}^{N} min^2\left\{\left(\frac{Ri}{N+1}\right)^{g(p)}, 1-\gamma\right\}Var[s(X_i)]$$

$$= \frac{1}{4}\sum_{i=1}^{N} min^2\left\{\left(\frac{Ri}{N+1}\right)^{g(p)}, 1-\gamma\right\}\frac{1}{N}$$

$$\xrightarrow{N\to\infty} \frac{1}{4}\int_{0}^{1} min^2\{u^{g(p)}, 1-\gamma\}\, du$$

$$= \frac{(1-\gamma)^2}{4}\left[1 - \left(\frac{2g(p)}{2g(p)+1}(1-\gamma)^{\frac{1}{g(p)}}\right)\right].$$

C. Tables

Table 1: Cases of the GLD

	λ_1	λ_2	λ_3	λ_4	α_3	α_4
Case 1	0	0.197454	0.134915	0.134915	0	3
Case 2	-0.116734	-0.351663	-0.13	-0.16	0.8	11.4
Case 3	0	-1	-0.1	-0.18	2.0	21.2
Case 4	3.586508	0.04306	0.025213	0.094029	0.9	4.2
Case 5	0	-1	-0.0075	-0.03	1.5	7.5
Case 6	0	1	0.00007	0.1	1.5	5.8

Table 2: Empirical Powers and Sizes of the Compared Tests

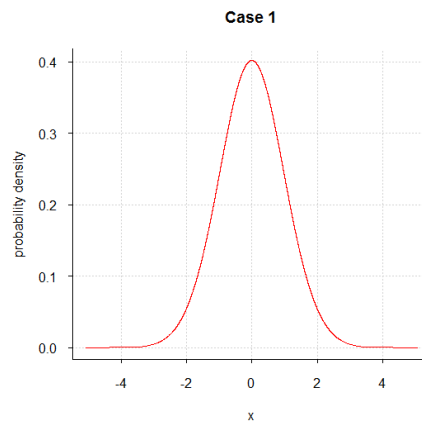| $\alpha=0.05$ Sampled distribution	θ	$\bar{W}$	$\bar{S}$	$\bar{B}$	$BW_1(\gamma)$ $\overline{BW}_1(0.1)$	$\overline{BW}_1(0.2)$	$\overline{BW}_1(0.4)$	$\overline{BW}_1(0.6)$	$\overline{BW}_1(0.8)$	$BW_2(\gamma)$ $\overline{BW}_2(0.1)$	$\overline{BW}_2(0.2)$	$\overline{BW}_2(0.4)$	$\overline{BW}_2(0.6)$	$\overline{BW}_2(0.8)$
Case 1	0	0.031	0.044	0.031	0.034	0.034	0.041	0.047	0.046	0.034	0.037	0.041	0.043	0.051
	0.2	0.269	0.217	0.251	0.247	0.250	0.249	0.246	0.237	0.257	0.253	0.261	0.259	0.247
	0.4	0.667	0.540	0.624	0.623	0.621	0.599	0.576	0.547	0.644	0.637	0.627	0.613	0.570
	0.6	0.927	0.833	0.890	0.886	0.884	0.870	0.857	0.843	0.911	0.903	0.884	0.874	0.854
	0.8	0.991	0.960	0.980	0.980	0.980	0.974	0.971	0.964	0.990	0.986	0.980	0.977	0.967
	1	1	0.997	1	1	1	0.999	0.999	0.997	1	1	1	1	0.997
	1.2	1	1	1	1	1	1	1	1	1	1	1	1	1
Case 2	0		0.061	0.060	0.061	0.060	0.056	0.059	0.061	0.059	0.059	0.063	0.061	0.061
	0.2		0.324	0.361	0.364	0.370	0.367	0.350	0.339	0.381	0.386	0.394	0.374	0.343
	0.4		0.756	0.816	0.807	0.800	0.783	0.773	0.763	0.840	0.841	0.829	0.804	0.773
	0.6		0.974	0.983	0.983	0.981	0.984	0.983	0.977	0.991	0.993	0.990	0.984	0.980
	0.8		0.994	0.999	0.999	0.997	0.994	0.994	0.994	0.999	0.999	0.999	0.997	0.994
	1		1	1	1	1	1	1	1	1	1	1	1	1
Case 3	0		0.050	0.054	0.056	0.054	0.054	0.054	0.051	0.060	0.057	0.059	0.057	0.054
	0.2		0.373	0.414	0.411	0.406	0.400	0.383	0.376	0.439	0.429	0.424	0.406	0.379
	0.4		0.834	0.857	0.854	0.850	0.846	0.844	0.840	0.877	0.881	0.863	0.847	0.843
	0.6		0.993	0.996	0.996	0.996	0.996	0.994	0.993	0.996	0.996	0.996	0.996	0.993
	0.8		1	1	1	1	1	1	1	1	1	1	1	1
	1		1	1	1	1	1	1	1	1	1	1	1	1
Case 4	0		0.059	0.069	0.067	0.067	0.064	0.059	0.060	0.079	0.077	0.071	0.063	0.060
	0.2		0.250	0.280	0.273	0.267	0.259	0.256	0.253	0.324	0.304	0.281	0.261	0.256
	0.4		0.657	0.709	0.700	0.686	0.671	0.661	0.659	0.746	0.727	0.699	0.680	0.663
	0.6		0.943	0.957	0.954	0.951	0.946	0.943	0.943	0.963	0.961	0.954	0.949	0.943
	0.8		1	1	1	1	1	1	1	1	1	1	1	1
	1		1	1	1	1	1	1	1	1	1	1	1	1
Case 5	0		0.051	0.057	0.056	0.054	0.053	0.051	0.053	0.056	0.056	0.054	0.053	0.053
	0.2		0.294	0.323	0.313	0.310	0.303	0.300	0.294	0.346	0.337	0.319	0.309	0.299
	0.4		0.756	0.777	0.773	0.764	0.761	0.756	0.756	0.801	0.790	0.776	0.764	0.756
	0.6		0.977	0.980	0.979	0.979	0.977	0.977	0.977	0.983	0.981	0.981	0.977	.977
	0.8		1	1	1	1	1	1	1	1	1	1	1	1
	1		1	1	1	1	1	1	1	1	1	1	1	1
Case 6	0		0.049	0.051	0.051	0.051	0.050	0.050	0.049	0.061	0.056	0.051	0.050	0.050
	0.2		0.279	0.284	0.280	0.280	0.280	0.279	0.279	0.311	0.300	0.280	0.280	0.279
	0.4		0.760	0.763	0.761	0.760	0.760	0.760	0.760	0.771	0.766	0.763	0.760	0.760
	0.6		0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
	0.8		1	1	1	1	1	1	1	1	1	1	1	1
	1		1	1	1	1	1	1	1	1	1	1	1	1

D. Figures

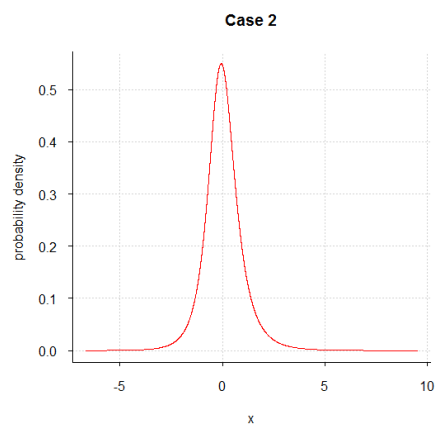

Figure 1: Density function for case 1 of the GLD

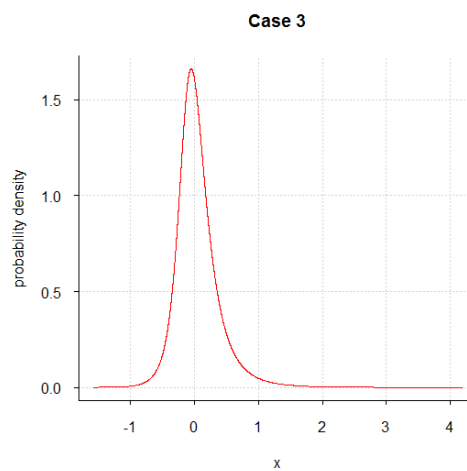Figure 2: Density function for case 2 of the GLD

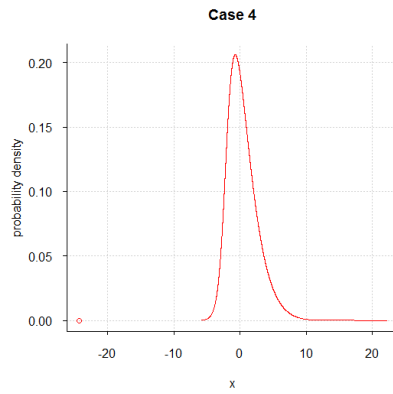Figure 3: Density function for case 3 of the GLD
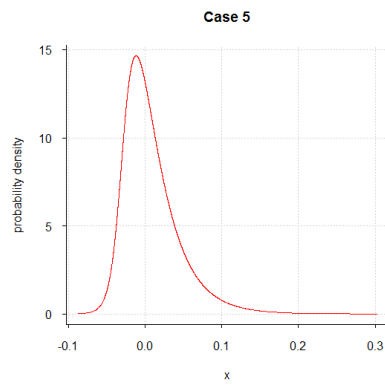
Figure 4: Density function for case 4 of the GLD
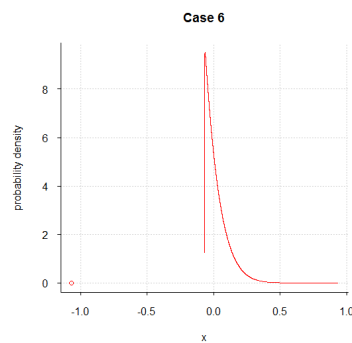

Figure 5: Density function for case 5 of the GLD

Figure 6: Density function for case 6 of the GLD