

Appendix. A. The Generalized Lambda Distribution (GLD)

The GLD is defined through its percentile function:

$$F^{-1}(y) = \lambda_1 + \frac{y^{\lambda_3} - (1 - y)^{\lambda_4}}{\lambda_2}, \quad (\text{A.1})$$

where $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are the location, scale, skewness and kurtosis parameters respectively. Its probability density function is given by:

$$f(x) = \frac{\lambda_2}{\lambda_3 y^{\lambda_3-1} + \lambda_4 (1 - y)^{\lambda_4-1}}, \quad \text{en } x = F^{-1}(y).$$

The regions in which the $\lambda_i, i = 1, 2, 3, 4$, yield valid distributions and explicit expressions for the moments, so as many other properties of the distribution can be find in Karian and Dudewicz 2000.

Appendix B. Proofs. Conditional Mean and Variance of the Proposed Test Statistic under H0.

Proof. Valid for $g(p) = p$ or $g(p) = \sqrt{p}$.

$$\begin{aligned} E[\overline{BW}|p] &= E \left[\frac{1}{N} \sum_{i=1}^N \min \left\{ \left(\frac{Ri}{N+1} \right)^{g(p)}, 1 - \gamma \right\} s(X_i) \mid p \right] \\ &= \frac{1}{N} \sum_{i=1}^N \min \left\{ \left(\frac{Ri}{N+1} \right)^{g(p)}, 1 - \gamma \right\} E[s(X_i)] \\ &= \frac{1}{2} \sum_{i=1}^N \min \left\{ \left(\frac{Ri}{N+1} \right)^{g(p)}, 1 - \gamma \right\} \frac{1}{N} \\ &\xrightarrow{N \rightarrow \infty} \frac{1}{2} \int_0^1 \min \{ u^{g(p)}, 1 - \gamma \} du \\ &= \frac{(1 - \gamma)}{2} \left[1 - \left(\frac{g(p)}{g(p) + 1} (1 - \gamma)^{\frac{1}{g(p)}} \right) \right], \end{aligned}$$

$$\begin{aligned}
N \text{Var}[\overline{BW} \mid p] &= N \text{Var} \left[\frac{1}{N} \sum_{i=1}^N \min \left\{ \left(\frac{Ri}{N+1} \right)^{g(p)}, 1 - \gamma \right\} s(X_i) \mid p \right] \\
&= \frac{N}{N^2} \sum_{i=1}^N \min^2 \left\{ \left(\frac{Ri}{N+1} \right)^{g(p)}, 1 - \gamma \right\} \text{Var}[s(X_i)] \\
&= \frac{1}{4} \sum_{i=1}^N \min^2 \left\{ \left(\frac{Ri}{N+1} \right)^{g(p)}, 1 - \gamma \right\} \frac{1}{N} \\
&\xrightarrow{N \rightarrow \infty} \frac{1}{4} \int_0^1 \min^2 \{u^{g(p)}, 1 - \gamma\} du \\
&= \frac{(1 - \gamma)^2}{4} \left[1 - \left(\frac{2g(p)}{2g(p) + 1} (1 - \gamma)^{\frac{1}{g(p)}} \right) \right].
\end{aligned}$$

C. Tables

Table 1: Cases of the GLD						
	λ_1	λ_2	λ_3	λ_4	α_3	α_4
Case 1	0	0.197454	0.134915	0.134915	0	3
Case 2	-0.116734	-0.351663	-0.13	-0.16	0.8	11.4
Case 3	0	-1	-0.1	-0.18	2.0	21.2
Case 4	3.586508	0.04306	0.025213	0.094029	0.9	4.2
Case 5	0	-1	-0.0075	-0.03	1.5	7.5
Case 6	0	1	0.00007	0.1	1.5	5.8

$\alpha = 0.05$					$BW_1 (\gamma)$					$BW_2 (\gamma)$				
Sampled distribution	Θ	$\hat{W}$	$\hat{S}$	$\hat{B}$	$\hat{BW}_1(0.1)$	$\hat{BW}_1(0.2)$	$\hat{BW}_1(0.4)$	$\hat{BW}_1(0.6)$	$\hat{BW}_1(0.8)$	$\hat{BW}_2(0.1)$	$\hat{BW}_2(0.2)$	$\hat{BW}_2(0.4)$	$\hat{BW}_2(0.6)$	$\hat{BW}_2(0.8)$
Case 1	0	0.031	0.044	0.031	0.034	0.034	0.041	0.047	0.046	0.034	0.037	0.041	0.043	0.051
	0.2	0.269	0.217	0.251	0.247	0.250	0.249	0.246	0.237	0.257	0.253	0.261	0.259	0.247
	0.4	0.667	0.540	0.624	0.623	0.621	0.599	0.576	0.547	0.644	0.637	0.627	0.613	0.570
	0.6	0.927	0.833	0.890	0.886	0.884	0.870	0.857	0.843	0.911	0.903	0.884	0.874	0.854
	0.8	0.991	0.960	0.980	0.980	0.980	0.974	0.971	0.964	0.990	0.986	0.980	0.977	0.967
	1	1	0.997	1	1	1	0.999	0.999	0.997	1	1	1	1	0.997
	1.2	1	1	1	1	1	1	1	1	1	1	1	1	1
Case 2	0		0.061	0.060	0.061	0.060	0.056	0.059	0.061	0.059	0.059	0.063	0.061	0.061
	0.2		0.324	0.361	0.364	0.370	0.367	0.350	0.339	0.381	0.386	0.394	0.374	0.343
	0.4		0.756	0.816	0.807	0.800	0.783	0.773	0.763	0.840	0.841	0.829	0.804	0.773
	0.6		0.974	0.983	0.983	0.981	0.984	0.983	0.977	0.991	0.993	0.990	0.984	0.980
	0.8		0.994	0.999	0.999	0.997	0.994	0.994	0.994	0.999	0.999	0.999	0.997	0.994
	1		1	1	1	1	1	1	1	1	1	1	1	1
	1.2		1	1	1	1	1	1	1	1	1	1	1	1
Case 3	0		0.050	0.054	0.056	0.054	0.054	0.054	0.051	0.060	0.057	0.059	0.057	0.054
	0.2		0.373	0.414	0.411	0.406	0.400	0.383	0.376	0.439	0.429	0.424	0.406	0.379
	0.4		0.834	0.857	0.854	0.850	0.846	0.844	0.840	0.877	0.881	0.863	0.847	0.843
	0.6		0.993	0.996	0.996	0.996	0.996	0.994	0.993	0.996	0.996	0.996	0.996	0.993
	0.8		1	1	1	1	1	1	1	1	1	1	1	1
	1		1	1	1	1	1	1	1	1	1	1	1	1
	1.2		1	1	1	1	1	1	1	1	1	1	1	1
Case 4	0		0.059	0.069	0.067	0.067	0.064	0.059	0.060	0.079	0.077	0.071	0.063	0.060
	0.2		0.250	0.280	0.273	0.267	0.259	0.256	0.253	0.324	0.304	0.281	0.261	0.256
	0.4		0.657	0.709	0.700	0.686	0.671	0.661	0.659	0.746	0.727	0.699	0.680	0.663
	0.6		0.943	0.957	0.954	0.951	0.946	0.943	0.943	0.963	0.961	0.954	0.949	0.943
	0.8		1	1	1	1	1	1	1	1	1	1	1	1
	1		1	1	1	1	1	1	1	1	1	1	1	1
	1.2		1	1	1	1	1	1	1	1	1	1	1	1
Case 5	0		0.051	0.057	0.056	0.054	0.053	0.051	0.053	0.056	0.056	0.054	0.053	0.053
	0.2		0.294	0.323	0.313	0.310	0.303	0.300	0.294	0.346	0.337	0.319	0.309	0.299
	0.4		0.756	0.777	0.773	0.764	0.761	0.756	0.756	0.801	0.79			

D. Figures

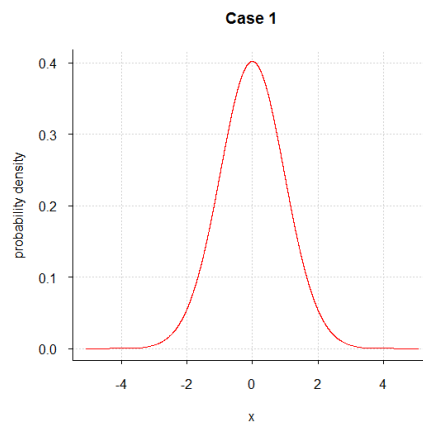


Figure 1: Density function for case 1 of the GLD

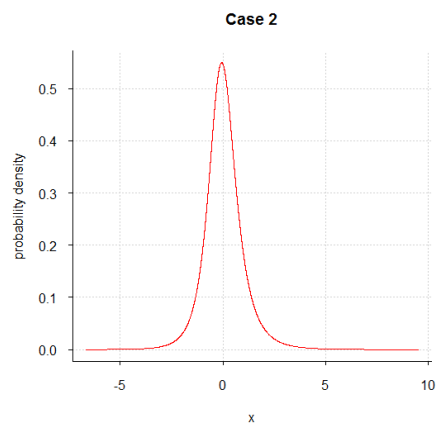


Figure 2: Density function for case 2 of the GLD

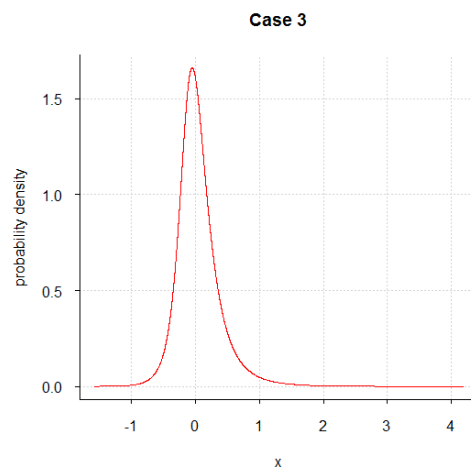


Figure 3: Density function for case 3 of the GLD

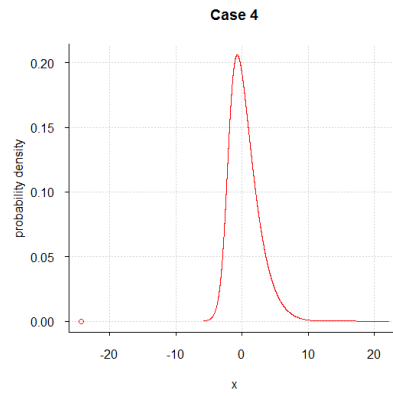


Figure 4: Density function for case 4 of the GLD

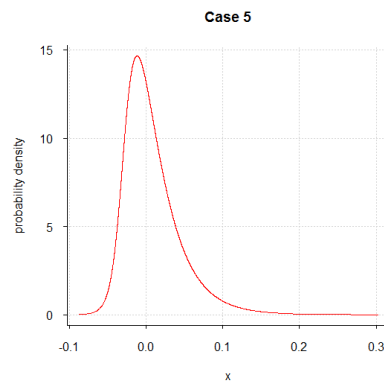


Figure 5: Density function for case 5 of the GLD

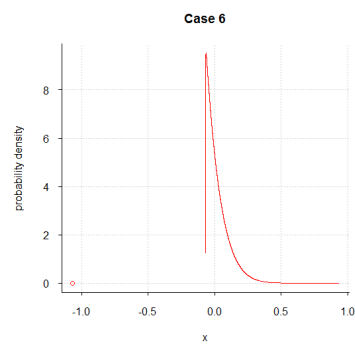


Figure 6: Density function for case 6 of the GLD