

# NON LINEAR TIME SERIES ANALYSIS OF THE EEG DURING SLEEP

por

David H. Campos<sup>1</sup>

## Resumen

**Campos, D.H.:** Non linear time series analysis of the EEG during sleep. Rev. Acad. Colomb. Cienc. **20(78):** 491-501. 1996. ISSN 0370-3908.

Se calculan la dimensión fractal y el máximo exponente de Lyapunov para el electroencefalograma (EEG) humano durante el sueño. Para este propósito se supone que el EEG es generado por un sistema dinámico determinista no lineal. Se encuentran pequeñas diferencias entre algunas etapas de sueño. El método usado se describe detalladamente e incluye cómo encontrar el espectro de exponentes de Lyapunov, la entropía de Kolmogorov-Sinai y la dimensión de Lyapunov. También se discuten resultados que contradicen el origen exclusivamente determinístico no lineal que algunos autores le atribuyen al EEG.

**Palabras claves:** EEG, dimensión, exponentes de Lyapunov, sistemas dinámicos deterministas no lineales, caos, series de tiempo, sueño.

## Abstract

I calculate the fractal dimension and the largest Lyapunov exponent for the human electroencephalogram (EEG) during sleep. For this purpose I assume that the EEG is generated by a deterministic nonlinear dynamical system. Slight differences among some sleep stages are found. The used method is thoroughly described and for completeness it includes how to find part of the Lyapunov exponents spectrum, Kolmogorov-Sinai entropy and Lyapunov dimension. I also discuss some results that apparently challenge the belief that the EEG is solely generated by a deterministic nonlinear dynamical system.

**Key words:** EEG, dimension. Lyapunov exponents, deterministic nonlinear dynamical systems, chaos, time series, sleep.

<sup>1</sup> Departamento de Ingeniería Eléctrica y departamento de Física. Universidad de los Andes.  
Apartado aéreo 53416, Bogotá, Colombia.

## 1. Introduction

The existence of deterministic chaos or low-dimensional nonlinear dynamics in the human EEG is still under discussion (Albano and Rapp 1992; Doyon 1992; Elbert et al. 1994; Fell et al. 1993; Frank et al. 1990; Palus 1992; Theiler 1995). Thus, the application of numerical techniques based on nonlinear systems to the EEG is an active research area. Although in this article I am particularly interested in describing clinical applications of nonlinear systems analysis, I will also discuss some results that dispute the presence of deterministic chaos in the EEG. Chaos is here regarded as a manifestation of some deterministic nonlinear dynamical systems that combine acute sensitivity to initial conditions with aperiodic and bounded behavior.

It is possible to start with a scalar time series for one observable, such as one EEG channel, and obtain a phase-space representation by phase-space reconstruction. Thus, information from all degrees of freedom coupled to the observable can be recovered. Although there is a variety of methods to achieve this (Casdagli et al. 1991; Landa and Rozenblyum 1989; Mindlin et al. 1991), the most popular and perhaps the only systematic procedure is time-delay embedding, originally described by Packard et al. (1980) and put on firmer footing by Mañé (1981) and Takens (1981). After having obtained a phase-space representation of the time series, we can perform a number of classifications, namely, dimensions (Elbert et al. 1994; Farmer et al. 1983; Grassberger 1983; Grassberger and Procaccia 1983a; Grassberger and Procaccia 1983c; Kantz and Schreiber 1994), Lyapunov exponents (Oseledec 1968), Kolmogorov-Sinai (KS) entropy (Farmer 1982; Grassberger and Procaccia 1983b), and spectrum of singularities  $f(\alpha)$  (Chhabra and Jensen 1989; Halsey et al. 1986).

Dimensions (fractal and of the natural measure) are the most basic property of an attractor<sup>2</sup>. An attractor is defined to be  $d$ -dimensional if in a neighborhood of every point it is diffeomorphic to an open subset of  $\mathbb{R}^d$ . A torus, for instance, has  $d=2$  because it opens into a two-dimensional rectangle. The attractor is strange if it is a fractal, i.e. its dimension is non-integer. The presence of a strange attractor almost always implies chaotic behavior. Dimensions provide the most basic level of knowledge necessary to characterize the properties of an attractor. They tell us the amount of information necessary for specifying

the position of a point on the attractor with a given precision, and are an inferior bound to the number of essential variables required to model the dynamics of the system. However, much more information about the dynamics of the system is provided by the spectrum of characteristic Lyapunov exponents. These exponents provide a quantitative measure of chaos by describing the mean rate of divergence of initially neighboring trajectories. In a chaotic system at least one exponent is positive, while for periodic or quasi-periodic behavior the largest exponent is zero. Furthermore, there are known relations between Lyapunov exponents and other measures, such as KS entropy and the dimension of the natural measure. Here I will analyze sleep EEG using both criteria: dimensions and Lyapunov exponents.

The fractal dimension of the attractor has been a common criterion to classify EEG signals. It has been found, for example, that in normal healthy subjects, the deeper the sleep, the lower the EEG dimensionality (Röschke and Aldenhoff 1992; Röschke and Aldenhoff 1993). Measurement of Lyapunov exponents of EEGs has been less frequent due to its technical difficulties. Frank et al. (1990) found the first Lyapunov exponent for an epileptic seizure, using a modification of the method by Wolf et al. (1985). The unmodified method was used by Babloyantz and Destexhe (1986) and by Fell et al. (1993) to determine the largest Lyapunov exponent during an instance of epilepsy and during sleep, respectively. Gallez and Babloyantz (1991) applied a procedure by Eckmann et al. (1986) that should find the complete spectrum of Lyapunov exponents and encountered at least two positive Lyapunov exponents in instances of alpha waves, deep sleep, and Creutzfeld-Jakob coma. This listing is by no means complete, but it gives an idea about recent research results.

In this article I look for changes in dimensions and Lyapunov exponents that may allow the characterization of sleep stages. Human sleep stages are classified as rapid-eye-movement (REM) (also fast-sleep or paradoxical sleep) and non-REM (slow sleep). The latter is further divided in stages 1–4. Stage 1 corresponds to drowsiness, 2 to light sleep, 3 to deep sleep, and 4 to very deep sleep. Although it is a fact that wakefulness EEG is richer in information than a sleep record, there are a number of conditions, especially in the domain of epileptic seizure disorders, in which sleep provides essential information. This obviously excludes the sleep disorders themselves. It is worth mentioning that 2 is the most informative sleep stage from a clinical point of view. (Niedermeyer 1993)

<sup>2</sup> Informally, an attractor is a set in phase-space that attracts trajectories after the transient has died out.

## 2. Procedure

In this section I outline in some detail the general analysis procedure performed on the data. Firstly, I assume that the data can be described by a deterministic flow of  $n$  generally coupled, nonlinear ordinary differential equations,

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}), \quad \mathbf{x} = [x_1(t), \dots, x_n(t)], \quad (1)$$

where  $\mathbf{f} = (f_1, \dots, f_n)$  are unknown functions of the coordinates  $\mathbf{x}(t)$ . Although this assumption has been very frequently used in prior investigations, it is still an active research area as I already mentioned. Now, using time-delay embedding (Mafé 1981; Packard et al. 1980; Takens 1981), I can reconstruct phase-space. Given the time series  $x(t_i)$  a  $d_E$ -dimensional phase portrait is reconstructed with delayed coordinates, i.e. a point on the attractor is given by

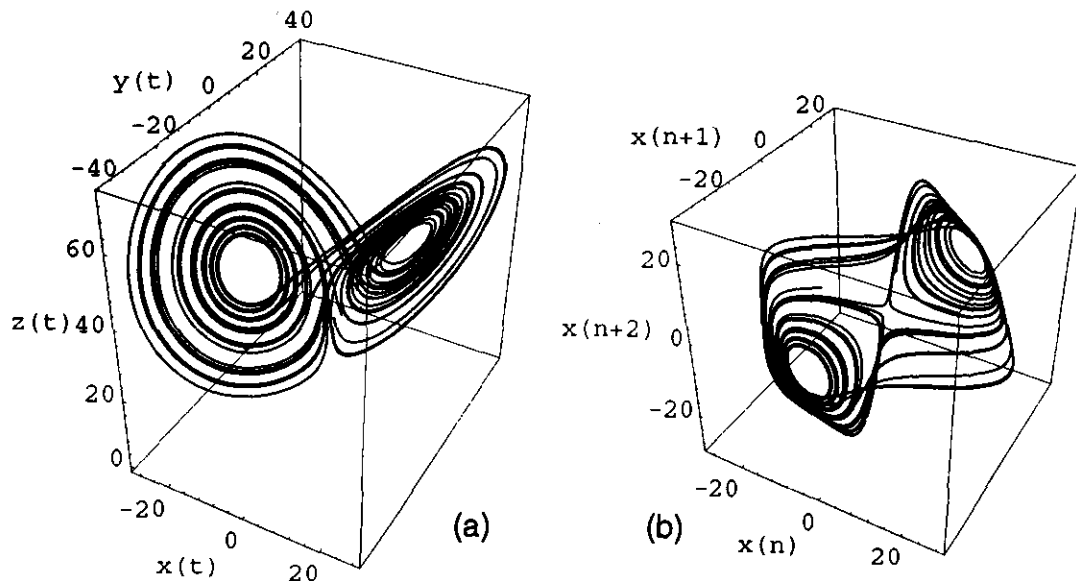
$$\mathbf{X}(t_i) = [x(t_i), x(t_i + \tau), x(t_i + 2\tau), \dots, x(t_i + (d_E - 1)\tau)], \quad (2)$$

where  $\tau$  is a multiple of the sampling time ( $T_s$ ), because we have a sampled signal. Thus, reconstruction requires finding an appropriate time-delay ( $\tau$ ) and embedding dimension ( $d_E$ ). According to Takens (1981), under some general conditions the orbit followed by  $\mathbf{X}(t)$  in this  $d_E$ -dimensional embedding space differs from the actual solution  $\mathbf{x}(t)$  of (1) only by a smooth change of coordinates.

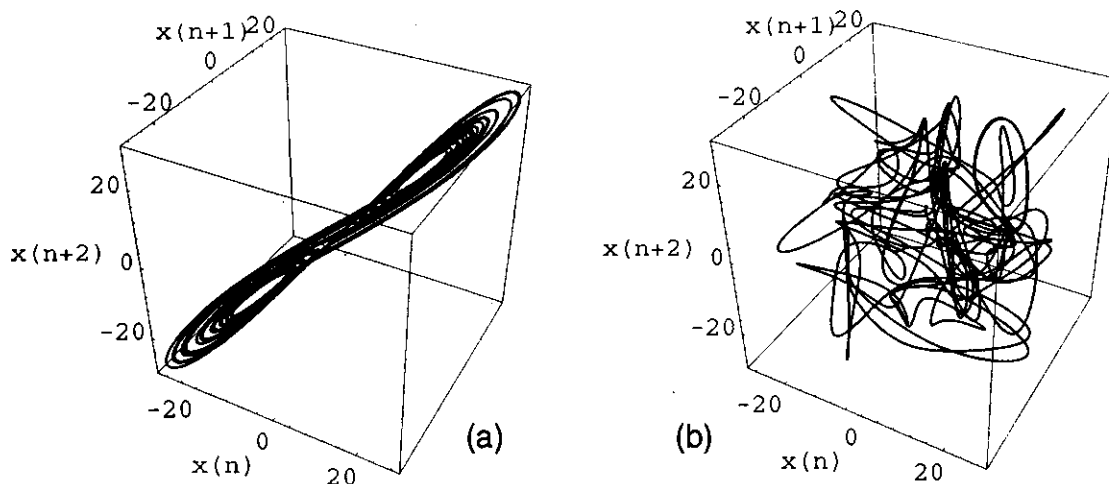
Mafé (1981) and Takens (1981) claim that phase-space reconstruction is independent of the time lag chosen. Nevertheless, this statement is not very useful for analyzing data. If  $\tau$  is chosen too small  $x(t)$  and  $x(t + \tau)$  will be so similar that they will not provide independent information. If, on the contrary,  $\tau$  is too large  $x(t)$  and  $x(t + \tau)$  will be totally uncorrelated, and the projection of the attractor will occur onto two totally unrelated directions. See figures 1 and 2 for an illustration of right and wrong choices of time-delay. So, to find an optimal  $\tau$  I use the method proposed by Fraser and Swinney (1986), which prescribes a selection based on the averaged mutual information (AMI) function ( $I$ ).  $I$  measures the number of bits that one does learn on the average about a set of measurements  $A=\{a_i\}$  from a set of measurements  $B=\{b_j\}$ . Its definition is

$$I(\tau) \equiv \sum_{a_i, b_j} P(a_i, b_j) \log_2 \left[ \frac{P(a_i, b_j)}{P(a_i) P(b_j)} \right],$$

where  $P(\cdot, \cdot)$  is the joint probability distribution for sets  $A$  and  $B$ , and  $P(\cdot)$  is the individual probability distribution for either of the measurements. In our case the two sets of measurements are  $x(t_i)$  and  $x(t_i + \tau)$ . So, since  $I$  tells us how much information one can learn about a measurement at



**Figure 1.** Phase-space representation of the Lorenz attractor (see table 1 for equations): (a) original attractor; (b) reconstructed attractor using time-delay embedding (time-delay = 0.1s corresponds to the first minimum of AMI).



**Figure 2.** Reconstructed Lorenz attractors: (a) time-delay ten times smaller than in Fig. 1b; (b) time-delay ten times larger than in Fig. 1b.

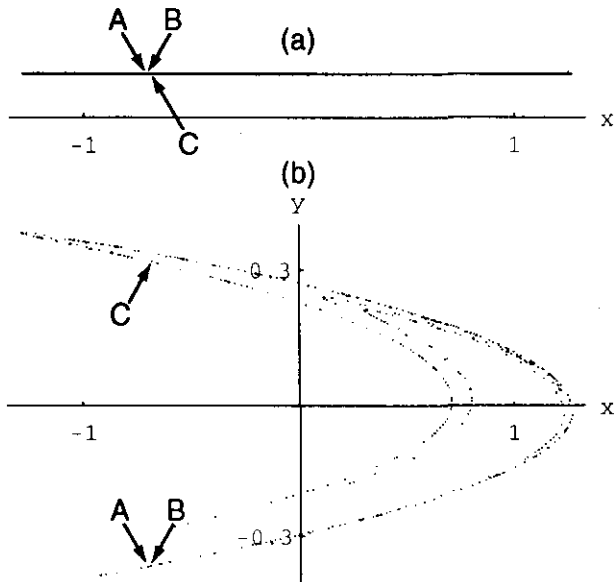
one time from a measurement taken at another time,  $\tau$  is chosen at the location of the first minimum of the AMI. When  $I$  does not have a minimum, but is a monotonously decreasing function, I use the empirical criterion proposed by Abarbanel et al. (1993), that selects  $\tau$  according to  $I(\tau)/I(0) \approx 1/5$ . If  $\tau \approx 10 \cdot T_s$  or more, the data *may* be oversampled according to Kennel and Abarbanel (1996). One way to deal with this problem is to down-sample (i.e. toss out data), until  $\tau \leq 5 \cdot T_s$  or so. This works, but it disposes of valuable hard won data in a quite casual fashion.

Takens (1981) proved that a *sufficient* condition for the embedding dimension is  $d_E > 2d_A$ , where  $d_A$  is the dimension of the attractor. Nevertheless, to obtain accurate estimation of exponents it is essential to find a *necessary* embedding dimension ( $d_N$ ), because too large an embedding dimension reduces the density of points defining the attractor, fills empty space with noise, and increases exponentially the computational cost. Furthermore,  $d_N$  puts an upper bound on the dimensionality of the system. To find  $d_N$  I use the method of false nearest neighbors (FNNs), originally described by Kennel et al. (1992), with one of the improvements (use of decorrelation intervals) proposed by Kennel and Abarbanel (1996). With this addition, the method not only improves FNN estimation, but provides means for testing for an extremely important feature: signal stationarity. When (2) is used to reconstruct phase-space, every point  $\mathbf{X}(t_i)$  will have another point that is its nearest neighbor ( $\mathbf{X}^{NN}(t_i)$ ), with nearness in the sense of some distance function (e.g. Euclidean distance). If these points are close together not because of the topology of the

attractor, but due to projection of the attractor in too low a dimensional space, then  $\mathbf{X}^{NN}(t_i)$  is a FNN of  $\mathbf{X}(t_i)$  (see figure 3 for an illustration). The procedure I apply then, is to find an estimation of the number of FNNs by gradually increasing embedding dimension. The embedding dimension for which percentage of FNNs approximately drops to zero is  $d_N$ . When the signal is distorted by noise the percentage of FNNs raises monotonously after reaching a minimum. If the value of the minimum is acceptable (i.e. it is close enough to zero) it can be chosen as  $d_N$ . It is worth mentioning that the complexity of an algorithm that looks for nearest neighbors in a straightforward manner is proportional to  $N^2/2$ , where  $N$  is the size of the data set. Thus, with increasing data sets a more efficient handling becomes obligatory. I use the box method described by Schreiber (1995) to tackle this problem ( $\sim N \log N$ ).

Once I have a phase-space reconstruction I apply the Grassberger and Procaccia (1983a) method as described by Holzfuss and Mayer-Kress (1986). Thus, I obtain an estimate of the correlation dimension ( $d_2$ ). Now, we can use the robust and now classical method of Wolf et al. (1985) to find the largest Lyapunov exponent ( $\lambda_1$ ) for every epoch. But in order to correctly estimate  $\lambda_1$  it is necessary to take into account some fundamental limitations of the procedure as expressed by Eckmann and Ruelle (1992). They have shown that to obtain correct results, the time series must satisfy

$$\log N > d_2 \log(D/r), \quad (3)$$



**Figure 3.** Data from the Hénon attractor (see table 1 for equations): (a) in too low an embedding space, (b) in a large enough embedding space. In (a) A, B, and C are neighbors, while in (b) it becomes clear that A is a true neighbor of B, while C is a false neighbor of B.

where  $N$  is the length of the time-series,  $d_2$  is the correlation dimension,  $D$  is the diameter of the reconstructed attractor, and  $r$  is a parameter of the Wolf et al. (1985) method. As a measure of  $D$  one may use the rms value of the time series

$$D^2 = \frac{1}{N} \sum_{i=1}^N [s(i) - \bar{s}]^2,$$

where  $\bar{s}$  is the average of the observations. In order to obtain sensible results it is required that  $r/D$  is of the order of 1%.

Then I apply the method proposed by Holzfuß and Lauterborn (1989) to find part of the characteristic Lyapunov exponents ( $\lambda_i$ ) spectrum (ordered by their magnitudes  $\lambda_1 > \lambda_2 > \lambda_3 > \dots$ ). Thus, the exponent obtained with the first method should help eliminating large spurious exponents that result from the application of the second one. I have not used the popular method by Eckmann et al. (1986) to find the Lyapunov exponents spectrum, due to concerns about its robustness expressed by Vastano and Kostelich (1986).

A relation between KS entropy and Lyapunov exponents was originally presented by Pesin (1977). Namely, KS entropy ( $K$ ) equals the sum of the positive Lyapunov

exponents if the system is ergodic<sup>3</sup>.  $K$  is the rate at which the system generates information, and is therefore inversely proportional to the predictability of the system. KS entropy is also a measure of how chaotic a system is.

The definition of Lyapunov dimension<sup>4</sup> (Young 1982) was introduced by Kaplan and Yorke (1979):

$$d_L = j + \frac{1}{|\lambda_{j+1}|} \sum_{i=1}^j \lambda_i,$$

where  $j$  is the largest integer for which  $\sum_{i=1}^j \lambda_i > 0$ . According to Farmer et al. (1983) for a typical attractor Lyapunov dimension should be equal to the dimension of the natural measure. Nevertheless, since the data sets we are working with are finite and not necessarily products of a perfectly stationary system, this equality does not hold exactly. Comparison of this dimension with the one obtained before is, nonetheless, a relatively accurate way of validating measured Lyapunov exponents.

In figure 4 I present a schematic summary of the procedure just described. Time sampling and epoch selection are briefly described in the next section.

### 3. Analyzed material and numerical results

Prior to EEG analysis I have tested the correctness of the above exposed method with simulated data: logistic mapping, Hénon mapping, and noiseless and noisy Lorenz system. For the latter I used additive gaussian distribution noise (zero mean and standard deviation equal to 1% of the reconstructed attractor size) as proposed by Yao and Tong (1994). Table 1 summarizes calculated values and references to sources of alternative results or alternative methods. In general, the values I have obtained depend strongly on the chosen parameters.

Figure 5 illustrates a noteworthy fact: for certain parameter values of the logistic map the FNN method gives erroneous results (FNN percentage never drops to zero and after reaching a minimum it rises as for a noisy time series). This is caused by the AMI method which yields too large a time delay (it should be 1). This is an interesting fact, because as I will discuss in some detail later I obtain similar results when I analyze certain sleep stages. With sleep stages, however, the fact that the FNN percentage does not approach zero is not due to a wrong time-delay selection.

<sup>3</sup> A system is ergodic if its Lyapunov exponents are independent of the position in phase-space.

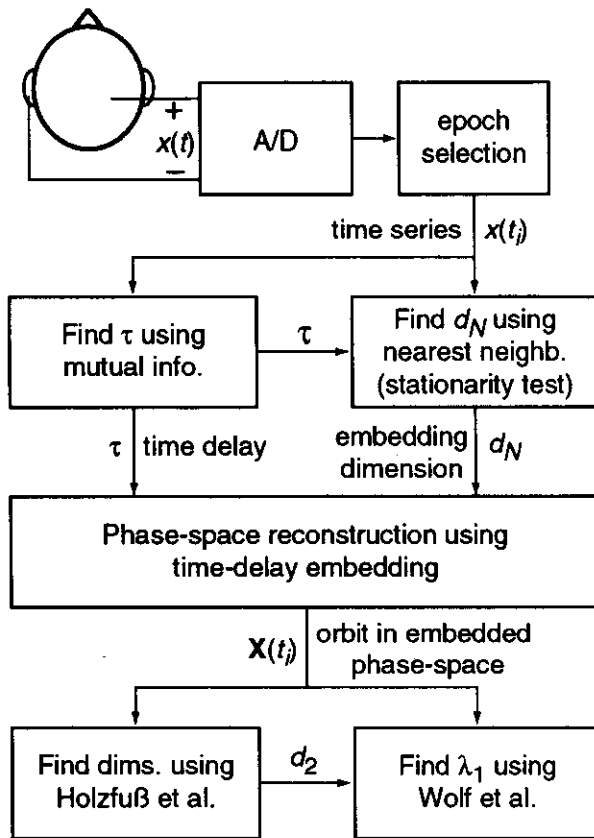


Figure 4. Summary of the applied method. See text for details.

Experimental data analysis has been performed on three EEG signals of different sleeping subjects taken from the MIT-BIH (1992) polysomnographic database. Subjects were monitored for evaluation of chronic obstructive sleep apnea syndrome (Broughton 1993). EEG recordings were taken in position C4–A1 according to the international 10–20 system (Reilly 1993) and had lengths that varied from two to six and a half hours, at a sampling rate of 250 samples per second. For analysis purposes, I have divided time series in 10,000 points epochs. Using such long sections is a must to comply with restriction (3). The 40-second epochs are selected visually in order to qualitatively assure stationarity, absence of artifacts, and permanence in a single sleep stage. Visual inspection also includes analysis of power spectra for shorter 512 points sub-epochs, as it is usual in clinical applications. The long epochs can be chosen, because I am further testing for stationarity using the method described by Kennel and Abarbanel (1996) (as I already discussed).

Unfortunately, I have not been able to analyze sleep stage 1, since its duration in the examined EEGs was too short. Furthermore, the distinction between deep drowsiness and light sleep is imprecise and has necessarily been drawn in a somewhat arbitrary and artificial manner.

Application of the AMI method (Fraser and Swinney 1986) yields time-delays in the range  $4 \cdot T_s - 9 \cdot T_s$ . Thus, in accordance to the criterion by Kennel and Abarbanel (1996) the signal is somewhat oversampled in most cases. Since the oversampling is not too acute, however, I do not down-sample the signal. Some AMI functions show marked minima, while others do not. In general I have found that the AMI function for REM sleep is the most jagged one, while the one for sleep stage 4 is the smoothest. Figure 6 shows two typical results of the AMI procedure.

As expected for a noisy signal, percentage of FNNs raises monotonously after reaching a minimum. For operative purposes if the minimum value of FNN percentage is below 5% I consider the epoch acceptable. I have not been able to find epochs corresponding to sleep stages 2 and REM that comply with this restriction. Figure 7 illustrates this fact. This issue is very important and will be further discussed in the next section.

Moreover, the FNN procedure further provides a stationarity test, by calculating the diverse results for various decorrelation intervals. For the procedure described herein I use decorrelation intervals in the  $0-20 \cdot \tau$  range. If the various plots coincide in what seems a unique bundle, the epoch is stationary. See figure 7 for an illustration.

The last part of the procedure is to find the dimension of the attractor and the largest Lyapunov exponent. Finding the correlation dimension is straightforward. The

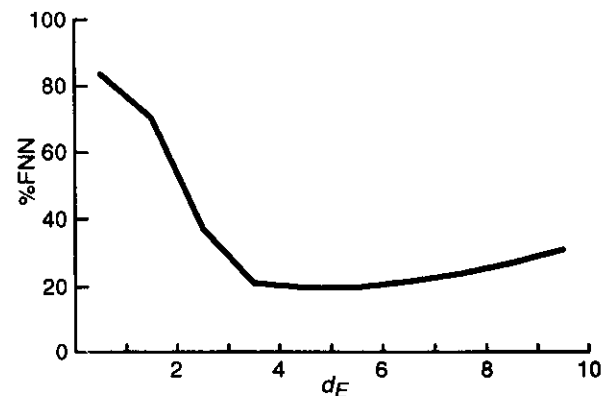


Figure 5. FNN percentage for the logistic mapping with  $a = 3.9$ . The procedure erroneously gives  $d_N > 1$ . Moreover the FNN percentage has a minimum that is well above zero and then rises as if the time series were noise contaminated.

System	Parameter values	Theoretical Lyapunov spectrum (bits/iter.)	$\tau$ using (Fraser and Swinney 1986) (iter.)	$d_N$ using (Kennel and Abarbanel 1996; Kennel et al. 1992)	$d_2$ using (Holzfuss and Mayer-Kress 1986)	Largest exponent using (Wolf et al. 1985)
<i>Logistic mapping:</i>						
(Binder and Campos 1996)	$a = 3.6$	$\lambda \approx 0.1814$	1	1	0.92	0.22
$x_{n+1} = a x_n(1 - x_n)$	$a = 3.9$	$\lambda \approx 0.491$	9	4	0.93	0.51
<i>Hénon iterator:</i>						
(Wolf et al. 1985)	$\begin{cases} a = 1.4 \\ b = 0.3 \end{cases}$	$\lambda_1 = 0.603$	8	2	1.22	0.56
$x_{n+1} = 1 - ax_n^2 + y_n$		$\lambda_2 = -2.34$				
$y_{n+1} = bx_n$						
<i>Noiseless Lorenz:</i>						
(Wolf et al. 1985)	$\begin{cases} \sigma = 16.0 \\ r = 45.92 \\ b = 4.0 \end{cases}$	$\lambda_1 = 2.16$	0.16s	3	2.08	2.03
$\dot{x} = \sigma(y - x)$		$\lambda_2 = 0.00$				
$\dot{y} = -xz + rx - y$		$\lambda_3 = -32.4$				
$\dot{z} = xy - bz$		(bits/s)				
(sampling rate: 100Hz)						
<i>Noisy Lorenz:</i>						
(Wolf et al. 1985)	Same as above	Same as above	0.16	3	2.10	2.13

Table 1. Results of procedure applied to simulated data.

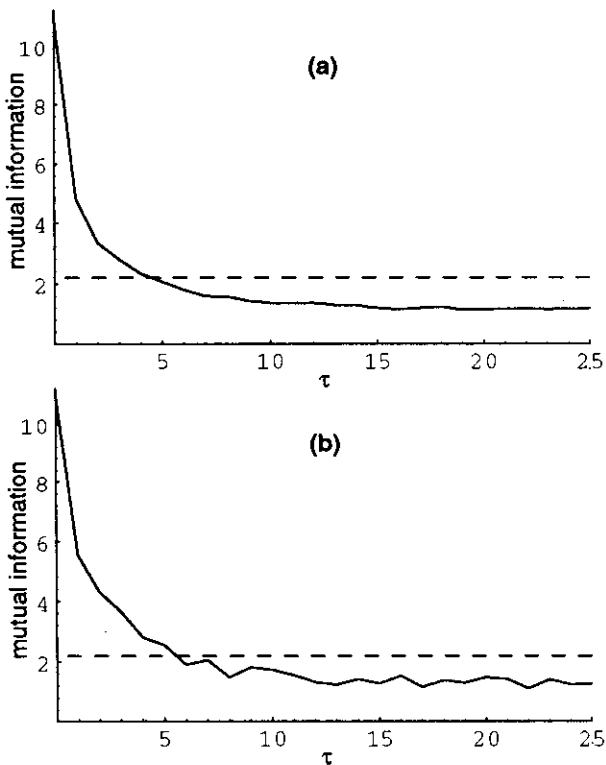
result of the Wolf et al. (1985) algorithm, on the other hand, is a series of points. In order to obtain a correct result, it is necessary that the points converge to a value like shown in figure 8. To achieve convergence a parameter (the evolution time between replacements) must be adjusted by trial and error. A wide range of evolution times must be checked, since we do not know the mechanism for chaos of the system. Table 2 is a summary of the obtained numerical results.

#### 4. Discussion and conclusions

Unfortunately it is not possible to find the dimensions and the largest Lyapunov exponent for every sleep stage. For the few sleep stages that can be analyzed, however, I have found that apparently the deeper the sleep, the lower the EEG dimensionality. This result is in agreement with the ones presented by Rösche and Aldenhoff (1992, 1993). The largest Lyapunov exponent also seems to decrease with deeper sleep. Variations among sleep stages are, nonetheless, not large.

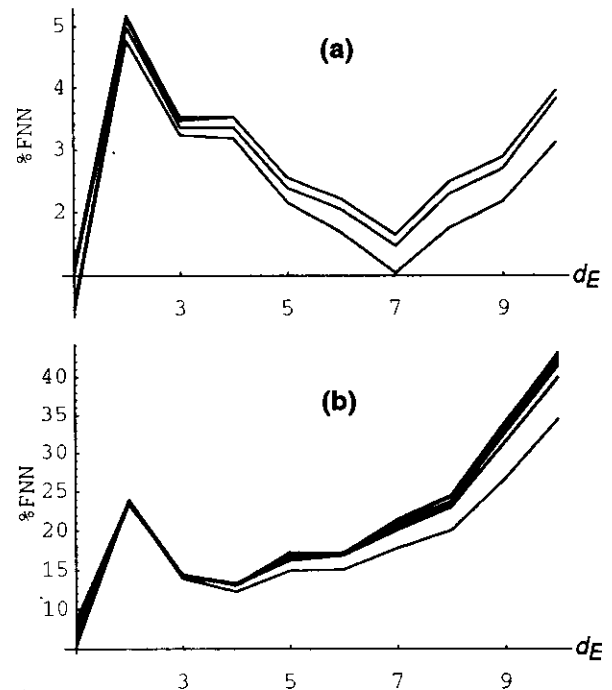
I have applied a procedure that finds the Lyapunov exponents spectrum. However this method does not yield valid results, due to a significant lack of robustness. Ideally the whole spectrum of Lyapunov exponents should be calculable, nevertheless, several factors can induce errors in the performed analysis: measurement conditions, nonstationarity of the signal (Elbert et al. 1994), spurious exponents (Grassberger 1991; Holzfuss and Lauterborn 1989), sampling-rates, time series and epochs lengths (Eckmann and Ruelle 1992; Kantz and Schreiber 1994), filtering, and noise reduction (Kantz and Schreiber 1994; Molinari and Dumermuth 1992). For a brief but complete discussion about influence of some of these factors specifically on the analysis of EEGs see Albano and Rapp (1992). Thus, we should call the measured exponents the "apparent spectrum of Lyapunov exponents". These have only meaning in a comparative sense.

The present EEG analysis has a number of improvements over previous investigations. Many previous researches have used the first zero-crossing of the auto-



**Figure 6.** Typical AMI functions for the EEG (solid line). The dashed line corresponds to 1/5 of the maximum AMI value. (a) Stage 4 sleep. No marked minimum is present. (b) REM sleep. In this case a first minimum is well defined. Notice the approximate agreement of the first minimum criterion and the 1/5 of maximum value criterion for this case.

correlation function (Abarbanel et al. 1993) to find the embedding time. This method, however, only assures linear independence of the used samples, which are most likely to be generated by a nonlinear system. For the analysis presented here I used the first minimum of the AMI function to find time-delay. Thus, the coordinates used are more generally independent and allow a better reconstruction



**Figure 7.** FNN percentage for the EEG. Curves correspond to different decorrelation intervals in the 0–20- $\tau$  range. They are bundled because both time series are almost stationary. (a) Stage 4 sleep. %FNN has minimum at  $d_E=7$ . Its value there is less than 5%, so  $d_N=7$  is acceptable. A fictitious rise in the 1–2 range is due to limitations of the box-algorithm (Schreiber 1995) for small  $d_E$ . (b) REM sleep. In this case the minimum occurs at  $d_E=4$ , and its corresponding value is too high to be acceptable. Compare this result with figure 5. As before, the 1–2 region is false.

of phase-space, as shown by Fraser and Swinney (1986).

Many previous investigations have not taken into account that the length of the selected epochs sets an upper limit to the numerical results one can get from the Wolf et al. (1985) method. In the procedure here presented I use the criterion proposed by Eckmann and Ruelle (1992) to avoid

Sleep stage	AMI	$\tau$	$d_N$	$d_2$ (avg.)	$\lambda_1$
1	—	—	—	—	—
2	jagged	5–9	—	—	—
3	smooth	4–6	5–6	3.38	0.5–0.7
4	smoothest	5–7	6–7	3.36	0.4–0.6
REM	most jagged	6–8	—	—	—

**Table 2.** Summary of the obtained numerical results.





**Figure 8.** Typical converging result of the Wolf et al. (1985) algorithm.

this limitation. Thus, long time series are required which make stationarity tests a must.

Another positive feature of the analysis presented here is the use of the FNN method (Kennel and Abarbanel 1996; Kennel et al. 1992) to find a necessary embedding dimension. This procedure directly addresses problems common in time-delay embedding, and thus works extremely well for phase-space reconstruction. I have also used the invariant saturation method described by Abarbanel et al. (1993), but I encountered that the FNN procedure is more robust. Furthermore, the FNN method gives a numerical estimate of the error that comes about from choosing too small an embedding dimension. This may be desirable at some times if we need to sacrifice some precision to lessen computing time. Since the method is quite accurate, we also obtain a good estimate of an upper bound to the dimensions of the attractor. Most recently Kennel and Abarbanel (1996) made appreciable improvements to this method (called now the method of FNN and false strands). I have already applied one of these improvements in this article, the use of a decorrelation interval, which not only improves FNN estimation, but is also a stationarity test.

As I mentioned before, figure 5 shows the erroneous result yielded by the FNN method when applied to the logistic mapping with  $a=3.9$  and  $\tau=9$ , where  $\tau$  has been found by the AMI method (Fraser and Swinney 1986). The FNN percentage does not approach zero and after reaching a minimum rises as if the orbit were noise contaminated. This outcome is not in accordance with the deterministic origin of the time series. The correct result is obtained when I use  $\tau=1$ . This illustrates that the method described herein must be applied carefully to orbits generated by discrete time systems.

For the EEGs of sleep stages 2 and REM (see figure 7b) I have found that the FNN percentage does not approach zero. In this case the result is not due to a wrong time-delay selection. Nor is noise responsible for this effect. I have also tested "deterministic" nonlinear systems with noisy parameters and have not found a similar result. Non-stationarity does not seem to be the reason for the effect either, since I obtain the same results for time series half as long. Thus, I presume that the most feasible explanation of the FNN not approaching zero is that the assumption that the EEG is generated by a deterministic nonlinear dynamical system is faulty. The FNN method (Kennel and Abarbanel 1996; Kennel et al. 1992) is directly founded on Takens' theorem (Takens 1981), which works for every deterministic nonlinear system. Thus, a FNN percentage that does not come close to zero tells us that there is no finite dimensional embedding space in which we can describe the system dynamics. This strongly suggests that the EEG does not originate solely from a deterministic nonlinear system. If this is true a totally different analysis approach must be applied to take into account the stochastic element of the EEG.

I have used a Cray J916 and a SUN workstation. Analysis of each 40-seconds epoch takes typically one and a half hours. The FNN method consumes most of the time (90% on the average). Therefore if this nonlinear analysis technique is to be used in real-time it is indispensable to use a more efficient method for finding near neighbors, than the one I have used for this analysis. I do not recommend circumventing this bottle-neck of the process by using a fixed embedding dimension, because the estimation of the largest Lyapunov exponent is very sensitive to its accurate selection. Furthermore, the FNN method provides information about stationarity that is indispensable because of the long epochs that are necessary. On the other hand, real-time Lyapunov analysis is being developed.

There are still a number of improvements and of additional investigations that can be performed based on the work done so far. One is to use some new methods developed for analysis of short noisy data sets. Thus we could analyze shorter time-series, making visual inspection of the data less critic and less necessary. Another open field is to study whether EEGs originate from low-dimensional deterministic nonlinear dynamics. In this article I have already mentioned a hint against this assumption, but it must be put on firmer ground. For instance, to test whether the EEG is colored noise is straightforward, using the new improved method of FNN and false strands (Kennel and Abarbanel 1996). It has the advantage that it needs much

less computation than the standard surrogate data approach used for this purpose. Furthermore, it distinguishes successfully low-dimensional chaos from noisy periodicity and other highly resonant linear systems that frequently fool surrogate data methods.

Articles on this subject rarely describe thoroughly the procedure followed. Decisive steps, such as choosing the necessary embedding dimension and time delay are usually described vaguely if not totally omitted. In this article I have made an effort to present the details of the procedure, to make EEG analysis by non-linear means a more accessible tool for everyone. Furthermore, the analysis procedure described herein is general and can be applied to any time series originating from a deterministic nonlinear dynamical system.

### 5. Acknowledgments

The author thanks very specially Prof. P.-M. Binder, Prof. D. Campos, Prof. J.C. Martínez, and Prof. J. Bohórquez for most valuable advice and discussion. I am also grateful to Prof. J. Holzfuß for making available his programs Lyapunov and Dimension and to Dr. F.W. Schneider for his program Mutinfo. And last but not least, I would like to thank the Centro MOX de Computación Avanzada and Centro de Microelectrónica CMUA, both at Universidad de los Andes, for the use of a Cray J916 and SUN workstations, respectively.

### 6. References

- Abarbanel, H.D., R. Brown, J.J. Sidorowich, & L.S. Tsimring. 1993. The analysis of observed chaotic data in physical systems. *Reviews of Modern Physics* **65**(4): 1331-1392.
- Albano, A.M. & P.E. Rapp. 1992. On the reliability of dynamical measures of EEG signals in *Jansen B.H. & M.E. Brandt* (ed.) Second annual conference on nonlinear dynamical analysis of the EEG. World Scientific Publishing, Houston. 117-139.
- Babloyantz, A. & A. Destexhe. 1986. Low-dimensional chaos in an instance of epilepsy. *Proc. Natl. Acad. Sci. USA* **83**: 3513-3517.
- Binder, P.-M. & D.H. Campos. 1996. Direct calculation of invariant measures for chaotic maps. *Physical Review E* **53**(5): R4259-4262.
- Broughton, R.J. 1993. Polysomnography: principles and applications in sleep and arousal disorders in *Niedermeyer E. & F.L.d. Silva* (ed.) *Electroencephalography: basic principles, clinical applications, and related fields*. Williams and Wilkins. Baltimore. 765-802.
- Casdagli, M., S. Eubank, J.D. Farmer, & J. Gibson. 1991. A theory of state space reconstruction in the presence of noise in *Atmanspacher H. & H. Scheingraber* (ed.) *Information dynamics*. Plenum Press. New York.
- Chhabra, A. & R.V. Jensen. 1989. Direct determination of the  $f(\alpha)$  singularity spectrum. *Physical Review Letters* **62**(12): 1327-1330.
- Doyon, B. 1992. On the existence and the role of chaotic processes in the nervous system. *Acta Biotheoretica* **40**: 113-119.
- Eckmann, J.P., S.O. Kamphorst, D. Ruelle, & S. Ciliberto. 1986. Liapunov exponents from a time series. *Physical Review A* **34**(6): 4971-4979.
- Eckmann, J.P. & D. Ruelle. 1992. Fundamental limitations for estimating dimensions and Lyapunov exponents in dynamical systems. *Physica D* **56**: 185-187.
- Elbert, T., W.J. Ray, Z.J. Kowalik, J.E. Skinner, K.E. Graf, & N. Birbaumer. 1994. Chaos and physiology: deterministic chaos in excitable cell assemblies. *Physiological Reviews* **74**(1): 1-47.
- Farmer, J.D. 1982. Information dimension and the probabilistic structure of chaos. *Zeitschrift für Naturforschung* **37a**: 1304-1325.
- Farmer, J.D., E. Ott, & J.A. Yorke. 1983. The dimension of chaotic attractors. *Physica D* **7**: 153-180.
- Fell, J., J. Röschke, & P. Beckmann. 1993. Deterministic chaos and the first positive Lyapunov exponent: a nonlinear analysis of the human electroencephalogram during sleep. *Biological Cybernetics* **69**: 139-146.
- Frank, G.W., T. Lookman, M.A.H. Nerenberg, C. Essex, J. Lemieux, & W. Blume. 1990. Chaotic time series analyses of epileptic seizures. *Physica D* **46**: 427-438.
- Fraser, A.M. & H.L. Swinney. 1986. Independent coordinates for strange attractors from mutual information. *Physical Review A* **33**(2): 1134-1140.
- Gallez, D. & A. Babloyantz. 1991. Predictability of human EEG: a dynamical approach. *Biological Cybernetics* **64**: 381-391.
- Grassberger, P. 1983. Generalized dimensions of strange attractors. *Physics Letters* **97A**(6): 227-230.
- Grassberger, P. 1991. Information and complexity measures in dynamical systems in *Atmanspacher H. & H. Scheingraber* (ed.) *Information Dynamics*. Plenum Press. New York. 15-33.
- Grassberger, P. & I. Procaccia. 1983a. Characterization of strange attractors. *Physical Review Letters* **50**(5): 346-349.

- Grassberger, P. & I. Procaccia.** 1983b. Estimation of the Kolmogorov entropy from a chaotic signal. *Physical Review A* **28**(4): R2591–2593.
- Grassberger, P. & I. Procaccia.** 1983c. Measuring the strangeness of strange attractors. *Physica D* **9**: 189–208.
- Halsey, T.C., M.H. Jensen, L.P. Kadanoff, I. Procaccia, & B.I. Shraiman.** 1986. Fractal measures and their singularities: the characterization of strange sets. *Physical Review A* **33**(2): 1141–1151.
- Holzfurt, J. & W. Lauterborn.** 1989. Liapunov exponents from a time series of acoustic chaos. *Physical Review A* **39**(4): 2146–2152.
- Holzfurt, J. & G. Mayer-Kress.** 1986. An approach to error-estimation in the application of dimension algorithms in Mayer-Kress G. (ed.) *Dimensions and entropies in chaotic systems.* Springer-Verlag. Berlin. 114–122.
- Kantz, H. & T. Schreiber.** 1994. Dimension estimates and physiological data. *Chaos* **5**(1): 143–154.
- Kaplan, J.L. & J.A. Yorke.** 1979. Numerical solution of a generalized eigenvalue problem for even mappings in Peitgen H.O. & H.O. Walter (ed.) *Functional Differential Equations and Approximation of Fixed Points.* Springer-Verlag. Berlin.
- Kennel, M.B. & H.D.I. Abarbanel.** 1996. False neighbors and false strands: a reliable minimum embedding dimension algorithm. **In print.**
- Kennel, M.B., R. Brown, & H.D.I. Abarbanel.** 1992. Determining embedding dimension for phase-space reconstruction using a geometrical construction. *Physical Review A* **45**(6): 3403–3411.
- Landa, P.S. & M.G. Rozenblyum.** 1989. Method for evaluating the embedding dimension of an attractor from experimental results. *Sov. Phys. Tech. Phys.* **34**(1): 6–10.
- Mañé, R.** 1981. On the dimension of the compact invariant sets of certain non-linear maps in Rand D. & L.-S. Young (ed.) *Dynamical Systems and Turbulence.* Lecture Notes in Mathematics. **898.** Springer. Berlin. 230–242.
- Mindlin, G.B., H.G. Solari, M.A. Natiello, R. Gilmore, & X.-J. Hou.** 1991. Topological analysis of chaotic time series data from the Belousov-Zhabotinskii reaction. *Journal of Nonlinear Science* **1**: 147–173.
- MIT-BIH** 1992. Polysomnographic Database. 2. ed., Biomedical Engineering Center at the Harvard-MIT Division of Health Sciences and Technology.
- Molinari, L. & G. Dumermuth.** 1992. Once more: dimension and Lyapunov exponents for the human EEG in Jansen B.H. & M.E. Brandt (ed.) *Second annual conference on nonlinear dynamical analysis of the EEG.* World Scientific Publishing. Houston. 140–155.
- Niedermeyer, E.** 1993. Sleep and EEG in Niedermeyer E. & F.L.d. Silva (ed.) *Electroencephalography: basic principles, clinical applications, and related fields.* Williams and Wilkins. Baltimore. 153–166.
- Oseledec, V.I.** 1968. A multiplicative ergodic theorem. Lyapunov characteristic numbers for dynamical systems. *Trans. Moscow Math. Soc.* **19**: 197–231.
- Packard, N.H., J.P. Crutchfield, J.D. Farmer, & R.S. Shaw.** 1980. Geometry from a time series. *Physical Review Letters* **45**(9): 712–716.
- Palus, M.** 1992. Testing for nonlinearity in the EEG in Jansen B.H. & M.E. Brandt (ed.) *Second annual conference on nonlinear dynamical analysis of the EEG.* World Scientific Publishing. Houston. 100–114.
- Pesin, Y.B.** 1977. Characteristic Lyapunov exponents and smooth ergodic theory. *Russian Math. Surveys* **32**(4): 55–114.
- Reilly, E.L.** 1993. EEG recording and operation of the apparatus in Niedermeyer E. & F.L.d. Silva (ed.) *Electroencephalography: basic principles, clinical applications, and related fields.* Williams and Wilkins. Baltimore. 104–124.
- Röschke, J. & J.B. Aldenhoff.** 1992. A nonlinear approach to brain function: deterministic chaos and sleep EEG. *Sleep* **15**(2): 95–101.
- Röschke, J. & J.B. Aldenhoff.** 1993. Estimation of the dimensionality of sleep-EEG data in schizophrenics. *European Archives of Psychiatry and Clinical Neuroscience* **242**: 191–196.
- Schreiber, T.** 1995. Efficient neighbor searching in nonlinear time series analysis. *International Journal of Bifurcation and Chaos* **5**(2): 349–358.
- Takens, F.** 1981. Detecting strange attractors in turbulence in Rand D. & L.-S. Young (ed.) *Dynamical Systems and Turbulence.* Lecture Notes in Mathematics. **898.** Springer. Berlin. 366–381.
- Theiler, J.** 1995. On the evidence for low-dimensional chaos in an epileptic electroencephalogram. *Physics Letters A* **196**: 335–341.
- Vastano, J.A. & E.J. Kostelich.** 1986. Comparison of algorithms for determining Lyapunov exponents from experimental data in Mayer-Kress G. (ed.) *Dimensions and entropies in chaotic systems.* Springer-Verlag. Berlin. 100–107.
- Wolf, A., J.B. Swift, H.L. Swinney, & J.A. Vastano.** 1985. Determining Lyapunov exponents from a time series. *Physica D* **16**: 285–317.
- Yao, Q. & H. Tong.** 1994. On prediction and chaos in stochastic systems. *Phil. Trans. R. Soc. Lond. A* **348**: 357–369.
- Young, L.-S.** 1982. Dimension, entropy and Lyapunov exponents. *Ergod. Th. & Dynam. Sys.* **2**: 109–124.