STELLAR MEMBERSHIP IN THE REGION OF THE DOUBLE CLUSTER *h* AND *x* PERSEI FROM PROPER MOTIONS AND POSITIONS USING A MULTIVARIATE PARAMETRIC MODEL. A FIRST APPROACH.

by

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Resumen

Uribe A., M. Higuera, and E. Brieva. Stellar membership in the region of the double cluster *h* and *x* Persei from proper motions and positions using a multivariate parametric model. A First Approach. Rev. Acad. Colomb. **Cienc. 22**(85): 475-484,1998. ISSN: 0370-3908.

Se estudia la región del cúmulo doble de Perseo utilizando movimientos propios y posiciones como variables cinemáticas, y se separan las estrellas de los cúmulos *h* y *x* de las del campo estelar. Se sigue un proceso de dos etapas en cada una de las cuales se superponen dos funciones de densidad normales bivariadas. Los parámetros del modelo se estiman resolviendo un sistema no lineal formado por nueve ecuaciones de verosimilitud, y se encuentran las probabilidades de pertenencia siguiendo un método Bayesiano (**Brieva** y **Uribe**, 1990). Se realizan dos estudios diferentes. En el primero se encuentran los miembros de *h* y de *x* usando un catálogo de 2820 estrellas publicado por (**Lavdovskij**, 1961). En la Figura 5 se muestra que hay una buena coincidencia entre las estrellas miembros que nosotros encontramos y aquellas que Lavdovskij considera como miembros probables o más probables usando otros criterios de pertenencia (**Lavdovskij**, 1965; **Van Maanen**, 1944; **Dieckvoss**, 1956). Un segundo estudio fue hecho trabajando con un catálogo de 5504 estrellas publicado por Muminov (**Muminov**, 1982, and http://adc.gsfc.nasa.gov/adc/authors). La Figura 6 muestra que hay una buena coincidencia entre las estrellas que Muminov considera como miembros probables con los miembros hallados en nuestro estudio de la región del cúmulo doble de Perseo.

Palabras claves: Cúmulos abiertos, pertenencia, cúmulo doble, movimientos propios, *h* y *x* Persei.

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Abstract

A segregation between cluster members and field stars in the region of the double cluster h and x Persei was done, working from proper motions and positions as kinematic random variables. A two stage process is followed overlapping each time two normal bivariate density functions. The model parameters are estimated solving a nine likelihood non linear equations system, and star membership probabilities are then found by a Bayesian approach (**Brieva** and **Uribe**, 1990). Two different studies are made. The first one finds the h and x Persei physical members working with a 2820 star catalogue published by Lavdovskij (**Lavdovskij**, 1961). Figure 5 shows a good agreement between the members segregated by our method and those stars considered by Lavdovskij as sure or as the most probable members using other memberhip sources (**Lavdovskij**, 1965; **Van Maanen**, 1944; **Dieckvos**, 1956). A secônd study is made working with a larger 5504 star catalogue published by Muminov (**Muminov**, 1982, and http://adc.gsfc.nasa.gov/adc/authors). Figure 6 shows a good agreement between the stars that Muminov considers as probable members and the members that we segregate in the region of the double cluster.

Key words: Open clusters, membership, double cluster, proper motions, h and x Persei.

Introduction

The double cluster h and χ Persei has been the subject of a very extensive analysis through the years (**Gahm** *et al*, 1970), and an early work on positions measurements was made by Lamonth since 1869 (**Alter** *et al*, 1970; **Lamont**, 1869). The two galactic clusters form a double system (**Oosterhoff**, 1937), and was **Struve** the first one to suggest in 1927 that the general high concentration of early stars in the nuclei of the double cluster may constitute a grouping in space (**Wildey**, 1964; **Struve**, 1927). The only indirect evidence for the physical connection between h and χ was given by **Pismis** in 1953 (**Tapia** *et al*, 1984). They are a pair of rich open clusters located at $l^{II} = 134°.63$ and $b^{II} = -3°.72$; and $l^{II} = 135°.08$ and $b^{II} = -3°.60$, with the following equatorial coordinates for the 1950 epoch: $\alpha = 2^h 15^m.5$ and $\delta = +56°55'$; $\alpha = 2^h 18^m.9$ and $\delta = +56°53'$.

The h grouping has also been called NGC869, Mellote 13 (Mel 13), Raab 9 (Rb 9) and Collinder 24 (Cr 24); χ persei has been named NGC 884, Mel 14, Rb 10 and Cr 25 (**Alter G.** *et al*). Investigations of many authors based mainly on astrophysical data show that the h and χ Persei clusters are surrounded by the association Perseus I, rich in hot giants and in an extensive number of blue and red supergiants (**Masevich**, 1957). These facts give a central role to the association Perseus I and the double cluster in the studies of stellar evolution.

The double cluster in Perseo is very well documented in the literature. **Oosterhoff**, 1937, made an extensive photographic study; **Bidelman**, 1943, **Johnson** and **Morgan**, 1955 and **Schild** *et al*, 1965, 1966, 1967, determined spectral types for cluster members; **Johnson** and **Morgan**, 1955, **Wildey** 1964, **Schild**, 1965, and **Moffat** and **Vogt**, 1974, made optical photometry in various systems, and **Mendoza** , 1967, and **Tapia** *et al*, 1984, worked infrared photometry for the clusters. Recent photometric works were made by **Denoyelle** *et al*, 1990, and by **Waelkens** *et al*, 1990. A study of chemical composition is found in **Dufton** *et al*, 1988. The clusters are situated close to the galactic equator in a milky way region of normal star density. This fact introduces a serious complication in the study of the physical properties of these clusters, as the relative number of foreground and background stars is consequently very large. The proper motions are of much importance as they may help us to distinguish the physical members of the clusters from the optical ones (**Oosterhoff**, 1937). In 1990 it was discovered, using Geneva photometry, that at least half of the brigther stars in h and χ Persei are variable stars, and that most of these variables are Be or related objects (**Waelkens** *et al*, 1990).

There is no agreement on the age and distance moduli of both clusters. On the basis of a photometric study, **Crawford** *et al* in 1970 concluded that both clusters have nearly the same age and a distance modulus $(m - M)_o$ close to 11.4 ± 0.4. **Balona** *et al*, 1984, adopted a distance modulus of 11.16 for the double cluster taking evolutionary effects into account. According to Schild the distances to h and χ are about 2150, and 2500 parsecs respectively (**Schild**, 1967). Sometimes it has been assumed that h Persei is younger than χ Persei on the basis of the occurrences of B0V stars in h, while the earliest type stars in χ Persei have and spectral classification of type B1 (**Tapia** *et al*), 1984.

The Hertzsprug-Russell diagram of the double cluster differs greatly from the diagram of the surrounding association. These differences have a very definite evolutionary significance (**Masevich**, 1956), and an accurate colormagnitude diagram for the brightest stars of the double cluster show that the reddening is not uniform. A comparison of Hertzsprug-Russell diagrams leads to the conclusion that the nucleus and the association are about the same age, but the formation of stars in the association continued after it had ceased in the nucleus (**Masevich** , 1956). Other groups of stars in the Galaxy are known to be as young, but no one so far investigated offers the possibility of such a large approximately homogeneous sample for study as the double cluster, together with its neighboring stars where early spectral types O5-B0 are found (**Wildey**, 1964).

The cluster χ is surrounded by supergiants five millon years older than other group of supergiants belonging to the h cluster and to an outer group of Perseus I centered between h and χ Persei (**Schild**, 1967). The association is seen proyected on the Perseus arm at around 3500 parsecs. This arm contains late type supergiants and consequently the probability that some of the supergiants refered to the h and χ persei association be background stars is non negligible (**Gahm, G.F** *et al*, 1970).

Proper motions, positions and radial velocities are the main membership criteria (**Gahm** *et al*, 1970). The purpose of this papers is to solve the membership problem in the region of h and χ Persei considered as a double cluster, working from proper motions and positions. Usually to discriminate members from non-members using proper motions and radial velocities is only partly successful if the maximum distance for which proper motions are detectable is somewhat shorter than the distance to the cluster (**Wildey**, 1963); that was the case for h and χ persei in the works made by Van Maanen, Machlin and others (**Van Maanen**, 1920 and 1944; **Macklin**, 1921). Things became somehow better for proper motions data using plates with grater difference epochs in the works made by Lavdoskij and Muminov (**Lavdoskij** 1961, 1965; **Muminov**, 1982, and http://adc.gsfc.nasa.gov/adc/authors).

These latter Russian sources giving positional and proper motions data have been the sources used in this paper.

Lavdovskij data

Lavdovskij published a catalog of proper motions and positions in the region of the double cluster h and χ Persei (**Lavdovskij**, 1961). His publication also covers eight other open clusters and their vicinities: NGC 129, NGC 457, NGC 581, NGC 752, NGC 1912, NGC 2168, NGC 6885, NGC 7092. The proper motions were determined by the film-to-film method using 32 pairs of plates taken with the Pulkovo normal astrograph. For the h and χ Persei clusters the plates are of very good quality and an epoch difference of 58 years; the worked field has 128 reference stars and covers an $80' \times 80'$ region with 2820 stars in the inner part, and a total of 3014 stars if those ones located in the outer region are considered (**Lavdovskij**, 1961). The research that he had begun in 1961 on the galactic clusters NGC 869 and NGC 884 is completed in a later study where he published what he considered as sure and as probable members of the h and χ Persei double cluster (**Lavdovskij**, 1965); he gives the Lavdoskij and the Oosterhoff identification star numbers and uses the membership determination made by Van Maanen in 1944 and Dieckvoss in 1956 (**Oosterhoff**, 1937; **A. Van Maanen**, 1944; **Dieckvoss**, 1956).

Muminov data

The results of an analysis of proper motions and UBV photometry for 5504 stars in the region of the double cluster h and χ persei are found in **Muminov**, 1982, and in the electronic address *http : //adc.gsfc.nasa.gov/ – adc/authors*. This is a photometric study and one of the latest reported work assigning new proper motions values in the region of the double cluster. In order to find them he used two pair of plates with an epoch difference of eighty years and obtained first the proper motions of 3086 stars brighter than 15.5 magnitudes in a circular region of 50 $'$ of radio centered in a point with equatorial coordintes given by $\alpha = 2^h 16^m.9$ and $\delta = +57°01'$ for the epoch 1950, and found then the proper motions of 1055 fainter stars but brighter than 17.0 magnitudes working in circular regions of 14 $'$ of radio centered in each one of the clusters. Proper motions were also found for 1363 stars in adjacent area using plates with an epoch difference of 38 years. These latter motions were reduced to the system of those in the cluster region using 214 stars in common. A catalogue with 1258 members of h and χ persei was published in his paper of 1982 (**Muminov**, 1982). He assigned membership probabilities with a criterion different to the Bayesian one that we use in this paper. His criterion is given by the equation

$$P_i = \exp\left\{-\frac{(\mu_{x_i} - \mu_{x_o})^2 + (\mu_{y_i} - \mu_{y_o})^2}{2\sigma^2}\right\}. \qquad (1)$$

When we applied his membership probability criterion to segregate stars in the region of the open cluster NGC 654, there were significative differences between the probabilities we found, and those probabilities assigned by other authors following a Bayesian probability approach usually employed when the vector point diagram of proper motions is modeled by the overlapping of two normal bivariate density functions (**Brieva** and **Uribe**, 1985; **Sanders**, 1971; **Latypov**, 1978). Besides, there is not a clear correlation between the high or low membership probabilities values he found, and his assignment to either cluster h or cluster χ (**Muminov**, 1982).

Membership in the Γ cluster from proper motions. The model

Let us define the Γ cluster,

$$\Gamma = h \cup \chi = \{s | s = Star,\ s \in h \ or\ s \in \chi\}, \tag{2}$$

as a mathematical union of the h or χ cluster members. The segregation between the Γ members and the field stars is carried out working from proper motions as ramdom variables, and on the basis of a mixed or contagious bivariate density function model given by,

$$\begin{aligned}\Phi(\mu_x,\mu_y) &= (1-n^f)\phi^\Gamma(\mu_x,\mu_y) + n^f\phi^f(\mu_x,\mu_y)\\ &= \frac{1-n^f}{2\pi(\sigma^\Gamma)^2}\exp\left\{-\frac{1}{2}\left[\left(\frac{\mu_x-\mu^\Gamma_{x_o}}{\sigma^\Gamma}\right)^2+\left(\frac{\mu_y-\mu^\Gamma_{y_o}}{\sigma^\Gamma}\right)^2\right]\right\}\\ &+\frac{n^f}{2\pi\sigma^f_x\sigma^f_y\sqrt{1-(\rho^f)^2}}*\\ &\exp\left\{-\frac{1}{2[1-(\rho^f)^2]}\left[\begin{array}{c}\left(\frac{\mu_x-\mu^f_{x_o}}{\sigma^f_x}\right)^2+\left(\frac{\mu_y-\mu^f_{y_o}}{\sigma^f_y}\right)^2\\ -2\rho^f\left(\frac{\mu_x-\mu^f_{x_o}}{\sigma^f_x}\right)\left(\frac{\mu_y-\mu^f_{y_o}}{\sigma^f_y}\right)\end{array}\right]\right\}\end{aligned} \tag{3}$$

In this equation, $\mu^\Gamma_{x_o}$, $\mu^\Gamma_{y_o}$, $\mu^f_{x_o}$, $\mu^f_{y_o}$ are the x and y components of the centroids of the cluster Γ and of the bivariate model for the field distribution; σ^Γ, σ^f_x, and σ^f_y, are the standard deviations of the cluster Γ and of the field stars distributions; n^f, n^Γ, are the percentage of field and cluster Γ stars; $1 - n^f = n^\Gamma$ is the proportion of cluster Γ members; ρ^f is the correlation coefficient of the normal bivariate field distribution. The mixed density function $\Phi(\mu_x, \mu_y)$ satisfies the normalization relation,

$$\iint \Phi(\mu_x,\mu_y)d\mu_x d\mu_y = 1. \tag{4}$$

The model is obtained by overlapping two normal bivariate density functions, an elliptic one for the field stars and a circular one for the cluster stars. It was first constructed by **Vasilevskis, Klemola and Preston** (1958) and has been widely used in astronomical works. The vector point diagram of proper motions or positional data of a considered sky region where a cluster is supposed to be, usually suggests its use, as it is clearly suggested by Figure 2, bidimensional plot for the proper motions given by Muminov (**Muminov,M.**,1982; http://adc.gsfc.nasa.gov/adc/authors).

The likelihood equations and the estimated parameters

The model parameters are the $\vec{\Theta}$-vector components,

$$\vec{\Theta} = (\mu^f_{x_o},\mu^f_{y_o},\sigma^f_x,\sigma^f_y,\mu^\Gamma_{x_o},\mu^\Gamma_{y_o},\sigma^\Gamma,n^f,\rho^f) \tag{5}$$

They are estimated by the maximum likelihood method, finding and solving firstly a nine non linear equations system, where each likelihood equation has been derived in association with the unknowns of the $\vec{\Theta}$-components. The likelihood equations that we found following this order are given by,

$$\sum_{i=1}^{N}\left\{\begin{array}{c}\left[\frac{n^f}{2\pi\sigma^f_x\sigma^f_y\sqrt{1-(\rho^f)^2}}\right]\Phi_i^{-1}A_i*\\ \left[\frac{\mu_{x_i}-\mu^f_{x0}}{(1-(\rho^f)^2)\left[\sigma^f_x\right]^2}-\frac{\rho^f(\mu_{y_i}-\mu^f_{y0})}{(1-(\rho^f)^2)\sigma^f_x\sigma^f_y}\right]\end{array}\right\}=0 \tag{6}$$

$$\sum_{i=1}^{N}\left\{\begin{array}{c}\left[\frac{n^f}{2\pi\sigma^f_x\sigma^f_y\sqrt{1-(\rho^f)^2}}\right]\Phi_i^{-1}A_i*\\ \left[\frac{\mu_{y_i}-\mu^f_{y0}}{(1-(\rho^f)^2)\left[\sigma^f_y\right]^2}-\frac{\rho^f(\mu_{x_i}-\mu^f_{x0})}{(1-(\rho^f)^2)\sigma^f_x\sigma^f_y}\right]\end{array}\right\}=0 \tag{7}$$

$$\sum_{i=1}^{N}\left\{\begin{array}{c}\left[\frac{n^f}{2\pi(\sigma^f_x)^2\sigma^f_y(1-(\rho^f)^2)^{3/2}}\right]\Phi_i^{-1}A_i*\\ \left[\left(\frac{\mu_{x_i}-\mu^f_{x0}}{\sigma^f_x}\right)^2-\frac{\rho^f(\mu_{x_i}-\mu^f_{x0})(\mu_{y_i}-\mu^f_{y0})}{\sigma^f_x\sigma^f_y}-1\right]\end{array}\right\}=0 \tag{8}$$

$$\sum_{i=1}^{N}\left\{\begin{array}{c}\left[\frac{n^f}{2\pi(\sigma^f_y)^2\sigma^f_x(1-(\rho^f)^2)^{3/2}}\right]\Phi_i^{-1}A_i*\\ \left[\left(\frac{\mu_{y_i}-\mu^f_{y0}}{\sigma^f_y}\right)^2-\frac{\rho^f(\mu_{x_i}-\mu^f_{x0})(\mu_{y_i}-\mu^f_{y0})}{\sigma^f_x\sigma^f_y}-1\right]\end{array}\right\}=0 \tag{9}$$

$$\sum_{i=1}^{N}\left[\frac{1-n^f}{2\pi(\sigma^\Gamma)^4}\right]\Phi_i^{-1}B_i\left\{\mu_{x_i}-\mu^\Gamma_{x0}\right\}=0 \tag{10}$$

$$\sum_{i=1}^{N}\left[\frac{1-n^f}{2\pi(\sigma^\Gamma)^4}\right]\Phi_i^{-1}B_i\left\{\mu_{y_i}-\mu^\Gamma_{y0}\right\}=0 \tag{11}$$

$$\sum_{i=1}^{N}\left\{\begin{array}{c}\left[\frac{1-n^f}{2\pi(\sigma^\Gamma)^3}\right]\Phi_i^{-1}B_i*\\ \left[\left(\frac{\mu_{x_i}-\mu^\Gamma_{x0}}{\sigma^\Gamma}\right)^2+\left(\frac{\mu_{y_i}-\mu^\Gamma_{y0}}{\sigma^\Gamma}\right)^2-2\right]\end{array}\right\}=0 \tag{12}$$

$$\sum_{i=1}^{N}\Phi_i^{-1}\left\{\frac{A_i}{2\pi\sigma^f_x\sigma^f_y\sqrt{1-(\rho^f)^2}}-\frac{B_i}{2\pi(\sigma^\Gamma)^2}\right\}=0 \tag{13}$$

$$\sum_{i=1}^{N}\left\{\begin{array}{c}\left[\frac{n^f}{2\pi\sigma^f_x\sigma^f_y(1-(\rho^f)^2)^{\frac{3}{2}}}\right]\Phi_i^{-1}A_i*\\ \left[\begin{array}{c}\rho^f+\frac{(1+(\rho^f)^2)(\mu_{x_i}-\mu^f_{x0})(\mu_{y_i}-\mu^f_{y0})}{(1-(\rho^f)^2)\sigma^f_x\sigma^f_y}\\ -\frac{\rho^f}{1-(\rho^f)^2}\left[\left(\frac{\mu_{x_i}-\mu^f_{x0}}{\sigma^f_x}\right)^2+\left(\frac{\mu_{y_i}-\mu^f_{y0}}{\sigma^f_y}\right)^2\right]\end{array}\right]\end{array}\right\}=0 \tag{14}$$

$$A_i = \left\{ \begin{array}{l} \exp\left\{ -\frac{1}{2[1-(\rho^f)^2]} \left[\left(\frac{\mu_{x_i} - \mu^f_{x0}}{\sigma^f_x} \right)^2 + \left(\frac{\mu_{y_i} - \mu^f_{y0}}{\sigma^f_y} \right)^2 \right] \right\} * \\ \exp\left\{ \frac{1}{[1-(\rho^f)^2]} \left[\rho^f \frac{(\mu_{x_i} - \mu^f_{x0})(\mu_{y_i} - \mu^f_{y0})}{\sigma^f_x \sigma^f_y} \right] \right\} \end{array} \right\} \tag{15}$$

$$B_i = \exp\left\{ -\frac{1}{2} \left[\left(\frac{\mu_{x_i} - \mu^{\Gamma}_{x0}}{\sigma^{\Gamma}} \right)^2 + \left(\frac{\mu_{y_i} - \mu^{\Gamma}_{y0}}{\sigma^{\Gamma}} \right)^2 \right] \right\} \tag{16}$$

$$\begin{aligned} \Phi_i &= \Phi(\mu_{x_i}, \mu_{y_i}) \\ &= (1 - n^f)\phi^{\Gamma}(\mu_{x_i}, \mu_{y_i}) + n^f \phi^f(\mu_{x_i}, \mu_{y_i}) \\ &= \frac{1-n^f}{2\pi(\sigma^2)^{\Gamma}} B_i + \frac{n^f}{2\pi\sigma^f_x\sigma^f_y\sqrt{1-(\rho^f)^2}} A_i \end{aligned} \tag{17}$$

$$\Phi_i^{-1} = \frac{1}{\Phi_i} \tag{18}$$

We have improved the appearence of the nine likelihood equations given them in terms of Φ_i^{-1}. It is worthy to underline that the previous equations were obtained without

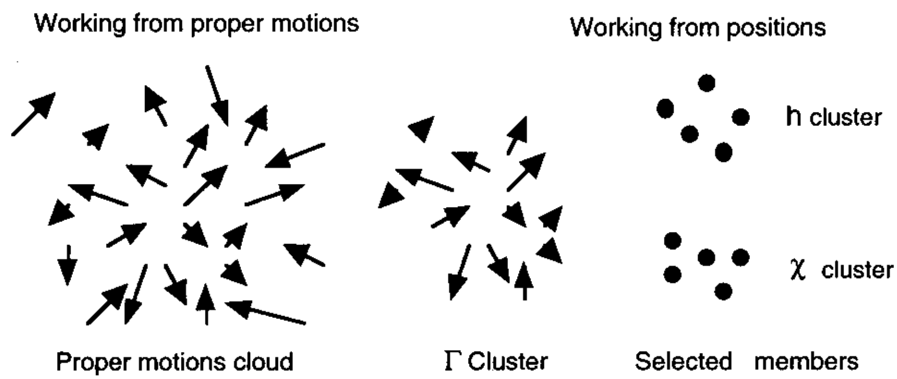

Figure 1. Representation of the two steps procedure to cast light on the way we get the h and the χ cluster members.

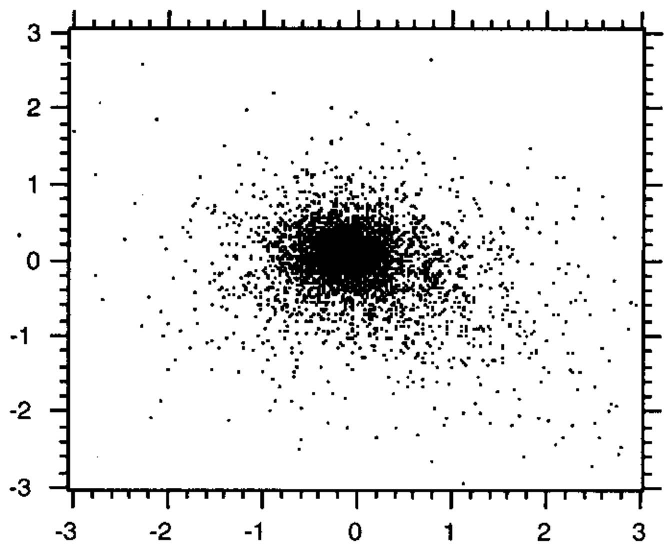

Figure 2. Nucleus and outer region in the Γ cluster from a vector point diagram of proper motions. The proper motions are given in 0.01 arcsec/century

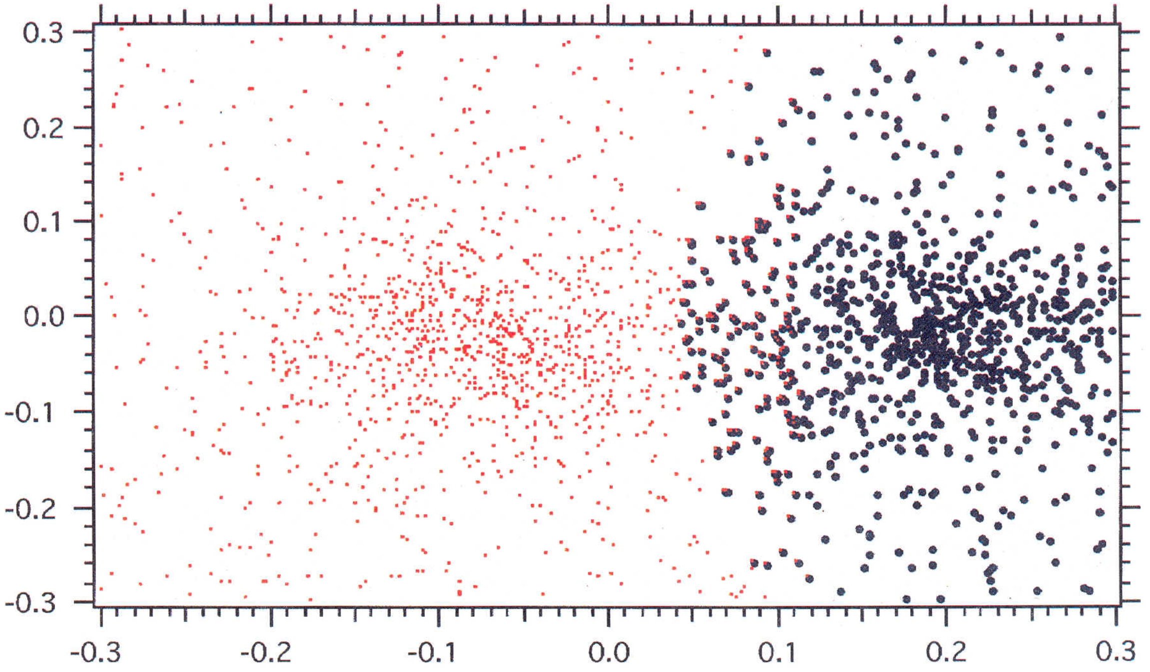

Figure 3. Positional vector point diagram of the segregated h cluster members (left side) and χ cluster members (right side). The coordinates x_i and y_i are positions on plate

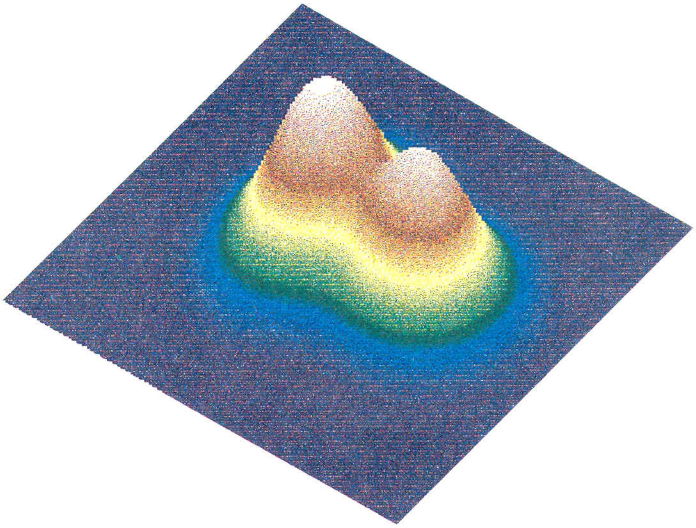

Figure 4. Positional mixture density function of the segregated h cluster members (left side) and χ cluster members (right side).

eliminating the correlation coefficient ρ^f following a previous approach (**Brieva** and **Uribe** , 1990). The eight equations currently used by other authors who perform a rotation to a preferential reference frame can be obtained by making the correlation coefficient ρ^f equal zero (**Sanders**, 1971; **Slovak**, 1977; **Zhao** *et al.*, 1990).

In a general way, for each star with proper motions components μ_{x_i}, μ_{y_i} or with positional x_i and y_i components, the membership probability of being a cluster member is given by the Bayes rule

$$P_i^{cluster} = \frac{w_{cluster}\Phi_i^{cluster}}{w_{cluster}\Phi_i^{cluster} + w_{field}\Phi_i^{field}} \qquad (19)$$

$w_{cluster}$ and w_{field} stand respectively for cluster and field weights. From these probabilities the stars in the region may be ascribed to the field or to the cluster using a suitable decision rule. The most usual but not the only one considers the i-star as a cluster member if $P_i^{cluster} \geq 0.50$ (Bayes minimum error rate of misclassification) (**Uribe** and **Brieva**, 1994).

Segregation of members using Lavdovskij data

In order to segregate the h cluster members from the χ members two steps have been followed, previously pruning, for each step, those outliers or data of the outer region. The first step segregates the Γ cluster members using only a proper motion model built with those 2820 Lavdovskij stars of the considered region, brighter than a 14.3 magnitude and with absolute proper motions little than 0.40. The previous 0.40 and 14.3 pruning level values are suggested by an analysis of the way in which Lavdovskij worked with the same data. In this way a file of 1650 stars, with probabilities $P_i^\Gamma = P_i^{h\cup\chi}$ greater than 0.65 of being Γ cluster members is found. In a second step this 1650 stars or Γ cluster is split in those h and χ cluster members, working now only from positional data. The Γ file to be split actually has only 1573 stars, once those stars with x_i or y_i positional components greater in absolute values tha 0.30 are pruned. This 0.30 level is sugested once again in the work by Lavdovskij. A Bayes minimum error rate of misclassification is used to assign membership positional probabilities: $P_i^\chi \geq 0.50$ *and* $P_i^h \geq 0.50$ (**Cabrera** and **Alfaro** 1990). A neat representation of this two steps procedure is given in figure 1 to cast light on the way we get the h and the χ cluster members. The already explained mixed model equation (3), the likelihood equations (6) to (14), and the probability criterion (19) were used in each one of the two steps of the process to achieve these segregations.

Thus in order to segregate the members of χ, using now positional data, we considered the positional cloud of the 1573 stars as a mixture of field stars and stars in the cluster χ. We apply once again our bivariate model using now positional data to decide which stars belong to the cluster χ with a P_i^χ probability value greater than a Bayesian level fixed in 0.40.

In order to find the h members we transformed the positional data (x,y) of the 1547 stars; instead of that cloud we use the (-x,y) symetyrical positional cloud that we see now as mixture of field and cluster h stars. Applying once again our bivariate model and fixing a Bayesian probability level in 0.40 we have segregated the h members.

In Figure 5, we have plotted the h (empty squares) and χ (empty circles) Persei cluster members that we found using in equation (19) a Bayesian probability membership level greater than 0.40. This figure allows a nice comparison between the stars that we found as members with those stars considered by Lavdovskij as probable P (filled red triangles and empty orange triangles) or most probable MP (filled green stars and empty blue stars) members, using other memberhip sources and criteria (**Lavdovskij**, 1961; **Van Maanen**, 1944; **Dieckvoss**, 1956). A high percentage of the stars considered as members in the work of Lavdovskij are also found as members in our study working with the explained two steps procedure. We consider that our result is in good agreement with the Lavdovskij work, a fact that highly motivated an application of our method to the more extensive Muminov catalogue as is explained in the next paragraph.

Segregation of members using Muminov data

The previous and already explained two steps method and the same bivariate models are now used to segregate the members of the h and χ clusters using Muminov data.

The most probable proper motions members in the considered region were found applying the previous first stage to the 5504 stars contained in the Muminov catalogue. The stars with absolute proper motions greater than 1.50 were pruned before solving the first step likelihood equations. A $P_i^\Gamma = P_i^{h\cup\chi}$ probability value greater than 0.70 was the fixed proper motion probability level or Bayes selection rule used to segregate the Γ cluster members. This cluster contains 2947 stars. The nucleus in figure 2 identifies this Γ cluster and is clearly distinguished from the outer region made out of the pruned field stars and those not chosen due to proper motion probabilities values little than 0.70.

In a forward second step these 2947 stars in the Γ cluster were split in 1385 h and 918 χ Persei cluster members working now from positions, pruning firstly those stars with x_i and y_i positional components greater in absolute value than 0.30 and using a probability level fixed in 0.40 as a Bayes minimum error rate of misclassification. The coordinates $x - i$ and y_i of these h and χ stars are plotted in figure 3. A tridimensional representation of these data is found in figure 4.

Figure 6 allows also a comparison between the stars that we identified as h (empty squares) and χ (empty circles) cluster members with those stars considered as h (filled red triangles) or χ (empty blue stars) members in the work of Muminov (**Muminov**, 1982). A detailed examination of Figure 6 reveals the good quality of our method to segregate the members of a double cluster using only proper motions and positions.

Discussion and Conclusions

We contribute with an original and non referenced method to solve the membership problem in a double cluster working from proper motions and positions as random variables. We agree in about an eighty per cent of the stars considered as sure or as most probable members by Lavdovskij using other memberhip sources and criteria (**Lavdovskij**, 1961; **Van Maanen**, 1944; **Dieckvoss**, 1956). We considered this agreement as a legitimate passport that led us to solve the membership problem using another 5504 stars catalogue of the double cluster in Perseus published by Muminov.

We also point out that the model and the likelihood equations we used when working either from proper motions or positions as random variables are also original contributions since the equations (6) to (14) were obtained without eliminating the correlation coefficient ρ in the model equation (3), and have a different algebraic formulation from the one we used in a previous published paper (**Uribe** and **Brieva**, 1994). These nine likelihood equations can be reduced to the eight equations currently used by other authors who perform a rotation to a preferential reference frame by making zero the correlation coefficient ρ. It is worthy of note that the method using rotation may be successful if the cluster and the field distributions overlap extensively (**Slovak**, 1977; **Cabrera** and **Alfaro**, 1985). Since this condition is not needed in our treatment, our likelihood equations have a general validity and lead to good results.

To solve the membership problem in a double cluster as we did for the double cluster in Perseus is to tackle a difficult problem; the cluster is situated closed to the galactic equator and so the relative number of foreground and background stars is consequently very large. Besides, both clusters, h and χ, overlap extensively (**Muminov, 1982**). Some few authors have made a membership analysis of this cluster during this century: Oosterhoff, working from Photometric data in 1937 (**Oosterhoff**, 1937); Van Maanen, from proper motions in 1944 without using something like the 1958 Vasilevskis model and the 1971 Sanders likelihood equations (**Van Maanen**, 1944; **Vasilevskis** *et al.*, 1958; **Sanders, W.**, 1971); Muminov, from proper motion and using some photometric criteria (**Muminov**, 1982). We have tried to contribute in this paper solving from proper motions and positions. Our work is tightly bound to a research in galactic open clusters that we began some years ago. The results of previous researches may be found in **Brieva** and **Uribe**, 1985, 1990, 1994, 1996 and **Uribe** and **Brieva**, 1994.

A fortran program called DOUBLECLUSTER has been developped to segregated between cluster and field stars. The maximun likehood equations were solved using the so called ZSYSTM subroutine kindly supplied by Doctor Mark Slovack some years ago. We will improve our segregation method using also photometric data in a new paper that we are working.

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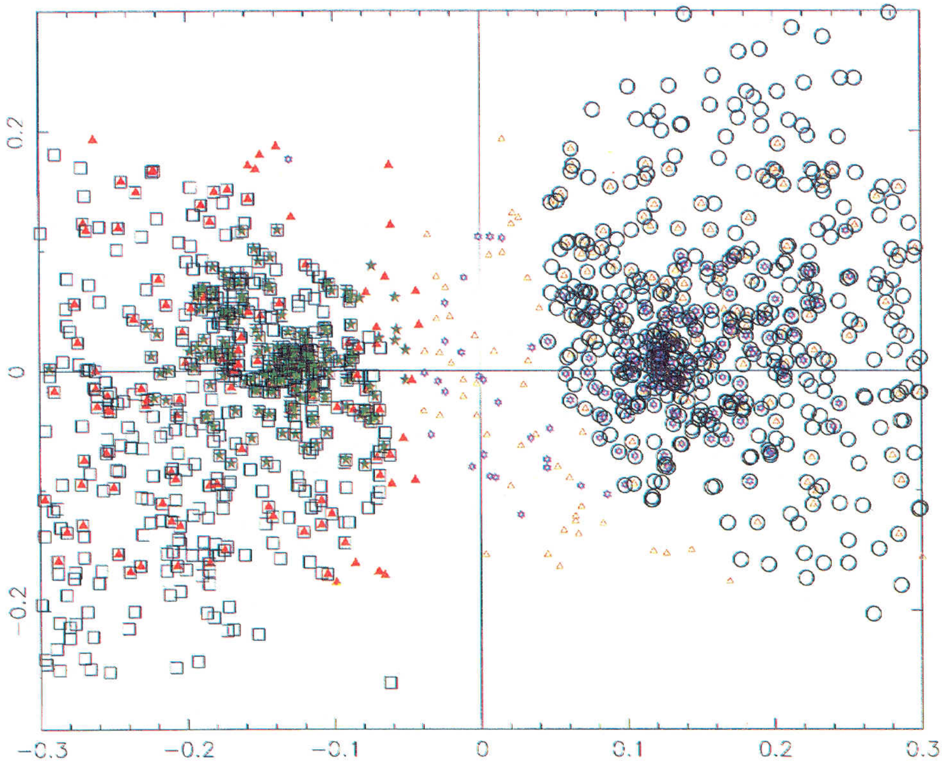

Figure 5. Stars segregated in the region of the double cluster by our method in comparison with the stars segregated by Lavdovskij. The meaning of the labels is explained in the text.

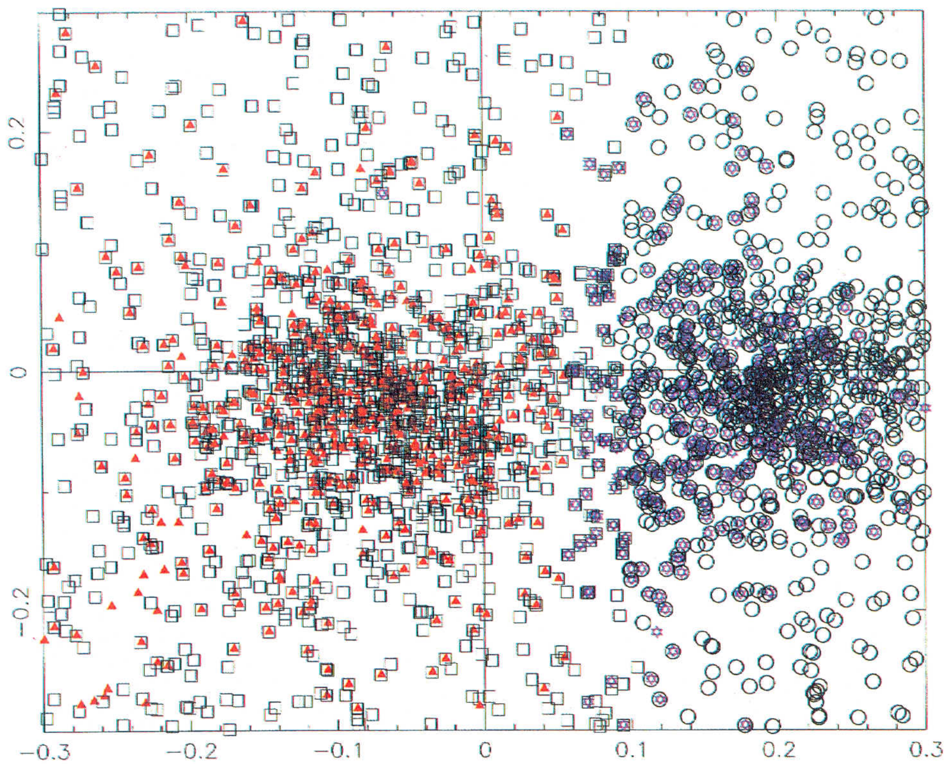

Figure 6. Stars segregated in the region of the double cluster by our method in comparison with the stars segregated by Muminov. The meaning of the labels is explained in the text.

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