NON-GAUSSIANITY AND LOOP CORRECTIONS IN A QUADRATIC TWO-FIELD SLOW-ROLL MODEL OF INFLATION. PART I

By

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Abstract

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I am showing in this paper that it is possible to attain very high, *including observable*, values for the level of non-gaussianity f_{NL} in a particular quadratic two-field slow-roll model of inflation with canonical kinetic terms. This is done by taking care of loop corrections both in the spectrum P_ζ and the bispectrum B_ζ of the primordial curvature perturbation ζ. Sizable values for f_{NL} arise even if ζ is generated during inflation. Five issues are considered when constraining the available parameter space: 1. we must ensure that we are in a perturbative regime so that the ζ series expansion, and its truncation, are valid. 2. we must apply the correct condition for the (possible) loop dominance in B_ζ and/or P_ζ. 3. we must satisfy the spectrum normalisation condition. 4. we must satisfy the spectral tilt constraint. 5. we must have enough inflation to solve the horizon problem.

Key words: Primordial curvature perturbation, non-gaussianity, slow-roll inflationary models.

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Resumen

Muestro en este artículo que es posible obtener valores altos, *incluso observables*, para el nivel de no gausianidad f_{NL} en un particular modelo inflacionario del tipo slow-roll con un potencial escalar cuadrático de dos componentes y términos cinéticos canónicos. Lo anterior se hace teniendo en cuenta correcciones de lazo tanto en el espectro P_ζ como en el biespectro B_ζ de la perturbación primordial en la curvatura ζ. Se obtienen valores grandes para f_{NL} incluso si ζ es generada durante inflación. Se tienen en cuenta cinco restricciones que reducen la ventana de parámetros disponible: 1. debemos estar seguros de estar trabajando en un régimen perturbativo de tal manera que la expansión en serie de ζ, y su truncamiento, sean válidas. 2. debemos aplicar la condición correcta acerca del (posible) dominio de las correcciones de lazo en B_ζ y/o P_ζ. 3. debemos satisfacer la condición de normalización del espectro 4. debemos satisfacer el índice espectral observado. 5. debemos asegurar el monto mínimo de inflación para resolver el problema de horizonte.

Palabras clave: Perturbación primordial en la curvatura, no gaussianidad, modelos inflacionarios del tipo slow-roll.

1 Introduction

The primordial curvature perturbation ζ (**Dodelson**, 2003; **Liddle & Lyth**, 2000; **Mukhanov**, 2005; **Weinberg**, 2008), and its δN expansion (**Lyth, Malik, & Sasaki**, 2005; **Lyth & Rodríguez**, 2005a; **Sasaki & Stewart**, 1996; **Starobinsky**, 1985), was the subject of study in two recent papers (**Cogollo, Rodríguez, & Valenzuela-Toledo**, 2008a; **Cogollo, Rodríguez, & Valenzuela-Toledo**, 2008b). The authors were interested in how well the convergence of the ζ series was understood, and if the traditional naive arguments to cut out the ζ series at second order (**Lyth & Rodríguez**, 2005a; **Zaballa, Rodríguez, & Lyth**, 2006), keeping only the tree-level terms to study the statistical descriptors of ζ (**Alabidi**, 2006; **Battefeld & Easther**, 2007; **Byrnes, Choi, & Hall**, 2008; **Byrnes, Sasaki, & Wands**, 2006; **Seery & Lidsey**, 2007; **Vernizzi & Wands**, 2006; **Yokoyama, Suyama, & Tanaka**; 2007; **Yokoyama, Suyama, & Tanaka**, 2008a; **Yokoyama, Suyama, & Tanaka**, 2008b), were reliable[3]. The authors argued that a previous study of the ζ series convergence, the viability of a perturbative regime, and the relative weight of the loop contributions against the tree-level terms, were completely necessary and in some cases surprising. For instance, the levels of non-gaussianity f_{NL} and τ_{NL} in the bispectrum $B_\zeta(k_1, k_2, k_3)$ and trispectrum $T_\zeta(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)$ of ζ respectively, for slow-roll inflationary models with canonical kinetic terms (**Liddle & Lyth**, 2000; **Lyth**, 2008; **Lyth & Riotto**, 1999), are usually thought to be of order $\mathcal{O}(\epsilon_i, \eta_i)$ (**Battefeld & Easther**, 2007; **Seery & Lidsey**, 2007; **Seery, Sloth, & Vernizzi**, 2008; **Vernizzi & Wands**, 2006; **Yokoyama, Suyama, & Tanaka**; 2007), were ϵ_i and η_i are the slow-roll parameters with $\epsilon_i, |\eta_i| \ll 1$ (**Lyth & Riotto**, 1999). However, in order to reach such a conclusion, only the tree-level terms were considered without giving a satisfactory explanation of why the loop contributions are comparatively suppressed, neither why ζ is indeed represented by the δN expansion, nor why the truncated δN expansion may be used. A couple of papers (**Alabidi**, 2006; **Byrnes, Choi, & Hall**, 2008) show that large, *and observable*, non-gaussianity in B_ζ is indeed possible for certain classes of *slow-roll* models with *canonical* kinetic terms and special trajectories in field space, relying only on the tree-level terms. Nonetheless, although the resultant phenomenology from these two papers is very interesting, a satisfactory argument about the tree-level dominance over the loop corrections was still lacking. In Refs. (**Cogollo, Rodríguez, & Valenzuela-Toledo**, 2008a; **Cogollo, Rodríguez, & Valenzuela-Toledo**, 2008b), the authors addressed these issues, and showed how important the requirements to guarantee the ζ series convergence and the existence of a perturbative regime are. Supported in Ref. (**Cogollo, Rodríguez, & Valenzuela-Toledo**, 2008a), I show in this paper that for a particular quadratic two-field *slow-roll* inflationary model with *canonical* kinetic terms, the one-loop correction to B_ζ might be much bigger than the tree-level terms, giving as a result large, *and observable*, non-gaussianity parameterised by f_{NL}. Based on the same grounds, the level of non-gaussianity τ_{NL} for the same slow-roll model studied here will be the subject of study in a companion paper (**Cogollo, Rodríguez, & Valenzuela-Toledo**, 2008c).

[3]In this paper, I follow the terminology of Ref. (**Byrnes, Koyama, Sasaki, & Wands**, 2007) to identify the tree-level terms and the loop contributions, within the δN formalism, in a diagrammatic approach. The associated diagrams are called *Feynman-like diagrams*.

2 Basic definitions and observation

Given the probability distribution function $f(\zeta)$, for the primordial curvature perturbation $\zeta(\mathbf{x}, t)$, there are an infinite number of standarized moments that work as statistical descriptors of $\zeta(\mathbf{x}, t)$:

$$\text{the mean value}: \; m_\zeta(1) \equiv \langle\zeta\rangle = \int \zeta f(\zeta) d\zeta \,, \quad (1)$$

$$\text{the variance}: \; m_\zeta(2) \equiv \int (\zeta - \langle\zeta\rangle)^2 f(\zeta) d\zeta \,, \quad (2)$$

$$\text{the skewness}: \; m_\zeta(3) \equiv \frac{\int (\zeta - \langle\zeta\rangle)^3 f(\zeta) d\zeta}{[m_\zeta(2)]^{3/2}} \,, \quad (3)$$

$$\text{the kurtosis}: \; m_\zeta(4) \equiv \frac{\int (\zeta - \langle\zeta\rangle)^4 f(\zeta) d\zeta}{[m_\zeta(2)]^2} \,, \quad (4)$$

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and so on.

Departures from the exact gaussianity come either from non-vanishing odd standarized moments $m_\zeta(n)$ with $n \geq 3$, in which case the probability distribution function is non-symmetric around the mean value, or from higher ($n \geq 4$) even standarized moments different to products of the variance, in which case the probability distribution function continues to be symmetric around the mean value although its "peakedness"[4] is bigger than that for a gaussian function, or from both of them.

Working in momentum space, the standarized moments of the probability distribution function have a direct connection with the correlation functions for the Fourier modes $\zeta_{\mathbf{k}} \equiv \int d^3k \zeta(\mathbf{x}) e^{-i\mathbf{k}\cdot\mathbf{x}}$ defined in flat space. As the n-point correlators of $\zeta_{\mathbf{k}}$ are generically defined in terms of spectral functions of the wavevectors involved[5]:

$$\text{two} - \text{point correlator} \rightarrow \text{spectrum } P_\zeta :$$
$$\langle\zeta_{\mathbf{k_1}}\zeta_{\mathbf{k_2}}\rangle \equiv (2\pi)^3 \delta^3(\mathbf{k_1} + \mathbf{k_2}) P_\zeta(k) \,, \quad (5)$$
$$\text{three} - \text{point correlator} \rightarrow \text{bispectrum } B_\zeta :$$
$$\langle\zeta_{\mathbf{k_1}}\zeta_{\mathbf{k_2}}\zeta_{\mathbf{k_3}}\rangle \equiv (2\pi)^3 \delta^3(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3}) \times B_\zeta(k_1, k_2, k_3) \,, \quad (6)$$
$$\text{four} - \text{point correlator} \rightarrow \text{trispectrum } T_\zeta :$$
$$\langle\zeta_{\mathbf{k_1}}\zeta_{\mathbf{k_2}}\zeta_{\mathbf{k_3}}\zeta_{\mathbf{k_4}}\rangle \equiv (2\pi)^3 \delta^3(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3} + \mathbf{k_4}) \times T_\zeta(\mathbf{k_1}, \mathbf{k_2}, \mathbf{k_3}, \mathbf{k_4}) \,, \quad (7)$$

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and so on,

the standarized moments of the distribution are then written in terms of momentum integrals of the spectral functions for the modes $\zeta_{\mathbf{k}}$:

$$\text{the variance}: m_\zeta(2) = \int \frac{d^3k}{(2\pi)^3} P_\zeta(k) \,, \quad (8)$$

$$\text{the skewness}: m_\zeta(3) = \frac{\int \frac{d^3k_1 \, d^3k_2}{(2\pi)^6} B_\zeta(k_1, k_2, k_3)}{\left[\int \frac{d^3k}{(2\pi)^3} P_\zeta(k)\right]^{3/2}} \,, \quad (9)$$

$$\text{the kurtosis}: m_\zeta(4) = \frac{\int \frac{d^3k_1 \, d^3k_2 \, d^3k_3}{(2\pi)^9} T_\zeta(\mathbf{k_1}, \mathbf{k_2}, \mathbf{k_3}, \mathbf{k_4})}{\left[\int \frac{d^3k}{(2\pi)^3} P_\zeta(k)\right]^2} \,, \quad (10)$$

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and so on.

Non-gaussianity in ζ is, therefore, associated with non-vanishing higher order spectral functions, starting from the bispectrum B_ζ.

Now I will parametrize the spectral functions of ζ in terms of quantities which are the ones for which observational bounds are given. Because of the direct connection between these quantities and the standarized moments of the probability distribution function $f(\zeta)$, I may also call these

[4]Higher even standarized moments different to products of the variance mean more of the variance is due to infrequent extreme deviations, as opposed to frequent modestly-sized deviations.

[5]Statistical homogeneity, i.e., invariance of the correlation functions under translations, requires the presence of the Dirac delta functions (**Ackerman, Carroll, & Wise**, 2007; **Carroll, Tseng, & Wise**, 2008; **Dimopoulos, Lyth, & Rodríguez**, 2008). Statistical isotropy, i.e., invariance of the correlation functions under rotations, requires that the spectrum P_ζ and bispectrum B_ζ are functions of the wavenumbers only (**Ackerman, Carroll, & Wise**, 2007; **Carroll, Tseng, & Wise**, 2008; **Dimopoulos, Lyth, & Rodríguez**, 2008). For the trispectrum T_ζ and the other higher order spectral functions, the momentum dependence also involves the direction of the wavevectors.

quantities as the statistical descriptors for $f(\zeta)$. The spectrum P_ζ is parametrized in terms of an amplitude $\mathcal{P}_\zeta^{1/2}$ and a spectral index n_ζ which measures the deviation from an exactly scale-invariant spectrum (**Liddle & Lyth**, 2000; **Weinberg**, 2008):

$$P_\zeta(k) \equiv \frac{2\pi^2}{k^3}\mathcal{P}_\zeta\left(\frac{k}{aH}\right)^{n_\zeta - 1}, \tag{11}$$

where a is the global expansion parameter and $H = \dot{a}/a$ is the Hubble parameter, with the dot meaning a derivative with respect to cosmic time. The bispectrum B_ζ and trispectrum T_ζ are parametrized in terms of products of the spectrum P_ζ, and the quantities f_{NL} and τ_{NL} respectively[6] (**Boubekeur & Lyth**, 2006; **Maldacena**, 2003):

$$B_\zeta(k_1,k_2,k_3) \equiv \frac{6}{5}f_{NL}\Big[P_\zeta(k_1)P_\zeta(k_2) + \text{cyclic permutations}\Big], \tag{12}$$

$$T_\zeta(\mathbf{k_1},\mathbf{k_2},\mathbf{k_3},\mathbf{k_4}) \equiv \frac{1}{2}\tau_{NL}\Big[P_\zeta(k_1)P_\zeta(k_2)P_\zeta(|\mathbf{k_1}+\mathbf{k_4}|) + \text{+cyclic permutations}\Big]. \tag{13}$$

Higher order spectral functions would be parametrized in an analogous way. Given the present observational state-of-the-art, n_ζ, f_{NL}, and τ_{NL} are the statistical descriptors that discriminate among models for the origin of the large-scale structure once $\mathcal{P}_\zeta^{1/2}$ has been fixed to the observed value. Since non-vanishing higher order spectral functions such as B_ζ and T_ζ imply non-gaussianity in the primordial curvature perturbation ζ, the statistical descriptors f_{NL} and τ_{NL} are usually called the levels of non-gaussianity.

The COBE satellite[7] provided us with a reliable value for the spectral amplitude $\mathcal{P}_\zeta^{1/2}$ (**Bunn & White**, 1997): $\mathcal{P}_\zeta^{1/2} = (4.957 \pm 0.094) \times 10^{-5}$ which is usually called the COBE normalisation. As regards the spectral index, the latest data release and analysis from the WMAP satellite[8] shows that $n_\zeta = 0.960 \pm 0.014$ (**Komatsu et. al.**, 2008) which rejects exact scale invariance at more than 2σ. Such a result has been extensively used to constrain inflation model building (**Alabidi & Lidsey**, 2008; **Alabidi & Lyth**, 2006), and although several classes of inflationary models have been ruled out through the spectral index, lots of models are still allowed; that is why it is so important an apropiate knowledge of the statistical descriptors f_{NL} and τ_{NL}. Present observations show that the primordial curvature perturbation ζ is almost, but not completely, gaussian. The level of non-gaussianity f_{NL} in the bispectrum B_ζ, after five years of data from NASA's WMAP satellite, is in the range $-9 < f_{NL} < 111$ at 2σ (**Komatsu et. al.**, 2008). There is at present no observational bound on the level of non-gaussianity τ_{NL} in the trispectrum T_ζ although it was predicted that COBE should either measure τ_{NL} or impose the lower bound $|\tau_{NL}| \lesssim 10^8$ (**Boubekeur & Lyth**, 2006; **Okamoto & Hu**, 2002). It is expected that future WMAP data releases will either detect non-gaussianity or reduce the bounds on f_{NL} and τ_{NL} at the 2σ level to $|f_{NL}| \lesssim 40$ (**Komatsu & Spergel**, 2001) and $|\tau_{NL}| \lesssim 2 \times 10^4$ (**Kogo & Komatsu**, 2006) respectively. The ESA's PLANCK satellite[9] (**The Planck Collaboration**, 2006), whose launch is currently scheduled in the spring of 2009, promises to reduce the bounds to $|f_{NL}| \lesssim 10$ (**Komatsu & Spergel**, 2001) and $|\tau_{NL}| \lesssim 560$ (**Kogo & Komatsu**, 2006) at the 2σ level if non-gaussianity is not detected. In addition, by studying the 21-cm emission spectral line in the cosmic neutral Hydrogen prior to the era of reionization, it is also possible to know about the levels of non-gaussianity f_{NL} and τ_{NL}; the 21-cm background anisotropies capture information about the primordial non-gaussianity better than any high resolution map of cosmic microwave background radiation: an experiment like this could reduce the bounds on the non-gaussianity levels to $|f_{NL}| \lesssim 0.2$ (**Cooray**, 2006; **Cooray, Li, & Melchiorri**, 2008), and $|\tau_{NL}| \lesssim 20$ (**Cooray, Li, & Melchiorri**, 2008) at the 2σ confidence. Finally, it is worth stating that there have been recent claims about the detection of non-gaussianity in the bispectrum B_ζ of ζ from the WMAP 3-year data (**Yadav & Wandelt**, 2008). Such claims, which report a rejection of $f_{NL} = 0$ at more that 2σ ($26.9 < f_{NL} < 146.7$), are based on the estimation of the bispectrum while using some specific foreground masks. The WMAP 5-year analysis (**Komatsu et. al.**, 2008) shows a similar behaviour when using those masks, but reduces the significance of the results when other more conservative masks are included allowing again the possibility of exact gaussianity.

[6]There is actually a sign difference between the f_{NL} defined here and that defined in Ref. (**Maldacena**, 2003). The origin of the sign difference lies in the way the observed f_{NL} is defined (**Komatsu & Spergel**, 2001), through the Bardeen's curvature perturbation (**Bardeen**, 1980): $\Phi^B = \Phi_L^B + f_{NL}(\Phi_L^B)^2$ with $\Phi^B = (3/5)\zeta$, and the way f_{NL} is defined in Ref. (**Maldacena**, 2003), through the gauge invariant Newtonian potential: $\Phi^N = \Phi_L^N + f_{NL}(\Phi_L^N)^2$ with $\Phi^N = -(3/5)\zeta$ (**Komatsu**, 2008).

[7]NASA's COBE mission homepage: http://lambda.gsfc.nasa.gov/product/cobe/.

[8]NASA's WMAP mission homepage: http://wmap.gsfc.nasa.gov/.

[9]ESA's PLANCK mission homepage: http://planck.esa.int/.

3 The model

According to the classification of inflationary models proposed in Ref. (**Dodelson, Kinney, & Kolb**, 1997), the small-field models are those of the form that would be expected as a result of spontaneous symmetry breaking, with a field initially near an unstable equilibrium point (usually taken to be at the origin) and rolling toward a stable minimum $\langle\phi\rangle \neq 0$. Thus, inflation occurs when the field is small relative to its expectation value $\phi \ll \langle\phi\rangle$. Some interesting examples are the original models of new inflation (**Albrecht & Steinhardt**, 1982; **Linde**, 1982), modular inflation from string theory (**Dimopoulos & Lazarides**, 2006), natural inflation (**Freese, Frieman, & Olinto**, 1990), and hilltop inflation (**Boubekeur & Lyth**, 2005). As a result, the inflationary potential for small-field models may be taken as

$$V = \sum_i \Lambda_i \left[1 - \left(\frac{\phi_i}{\mu_i}\right)^p\right] , \qquad (14)$$

where the subscript i here denotes the relevant quantities of the ith field, p is the same for all fields, and Λ_i and μ_i are the parameters describing the height and tilt of the potential of the ith field.

While Ref. (**Ahmad, Piao, & Quiao**, 2008) studies the spectrum of ζ for general values of the parameter p and an arbitrary number of fields, assuming ζ series convergence and tree-level dominance, I will specialize to the $p = 2$ case for two fields ϕ and σ:

$$V = V_0\left(1 + \frac{1}{2}\eta_\phi\frac{\phi^2}{m_P^2} + \frac{1}{2}\eta_\sigma\frac{\sigma^2}{m_P^2}\right) , \qquad (15)$$

where I have traded the expressions

$$\Lambda_1 + \Lambda_2 \quad \text{for} \quad V_0 , \qquad (16)$$

$$\frac{\Lambda_1}{\mu_1^2} \quad \text{for} \quad -V_0\frac{\eta_\phi}{2m_P^2} , \qquad (17)$$

and

$$\frac{\Lambda_2}{\mu_2^2} \quad \text{for} \quad -V_0\frac{\eta_\sigma}{2m_P^2} , \qquad (18)$$

and defined m_P as the reduced Planck mass. On doing this, and assuming that the first term in Eq. (15) dominates, $\eta_\phi < 0$ and $\eta_\sigma < 0$ become the usual η slow-roll parameters associated with the fields ϕ and σ.

I have chosen for simplicity the $\sigma = 0$ trajectory (see Fig. 1) since in that case the potential in Eq. (15) reproduces for some number of e-folds the hybrid inflation scenario (**Linde**, 1994) where ϕ is the inflaton and σ is the waterfall field. Non-gaussianity in the bispectrum B_ζ of ζ for such a model has been studied in Refs. (**Alabidi**, 2006; **Byrnes, Choi, & Hall**, 2008; **Cogollo, Rodríguez, & Valenzuela-Toledo**, 2008a; **Enqvist & Väihkönen**, 2004; **Lyth & Rodríguez**, 2005a; **Lyth & Rodríguez**, 2005b; **Väihkönen**, 2005; **Zaballa, Rodríguez, & Lyth**, 2006); in particular, Ref. (**Cogollo, Rodríguez, & Valenzuela-Toledo**, 2008a) shows that the one-loop correction dominates over the tree-level terms, generating in this way large values for f_{NL} even if ζ *is* generated during inflation. Refs. (**Alabidi**, 2006; **Byrnes, Choi, & Hall**, 2008), in contrast, work only at tree-level with the same potential as Eq. (15) but relaxing the $\sigma = 0$ condition, finding that large values for f_{NL} are possible for a small set of initial conditions. Ref. (**Cogollo, Rodríguez, & Valenzuela-Toledo**, 2008b) studies the trispectrum T_ζ of ζ in this model considering dominant loop corrections with $\sigma = 0$; its results show that large values for τ_{NL} are generated even if ζ *is* generated during inflation.

The slow-roll conditions for single-field inflationary models with canonical kinetic terms read

$$\dot\phi^2 \ll V(\phi) , \qquad (19)$$

$$|\ddot\phi| \ll |3H\dot\phi| , \qquad (20)$$

where ϕ is the inflaton field and $V(\phi)$ is the scalar field potential. On defining the slow-roll parameters ϵ and η_ϕ as (**Liddle & Lyth**, 2000)

$$\epsilon \equiv -\frac{\dot H}{H^2} , \qquad (21)$$

$$\eta_\phi \equiv \epsilon - \frac{\ddot\phi}{H\dot\phi} , \qquad (22)$$

the slow-roll conditions in Eqs. (19) and (20) translate into strong constraints for the slow-roll parameters: $\epsilon, |\eta_\phi| \ll 1$. Multifield slow-roll models may also be characterized by a set of slow-roll parameters which generalize those in Eqs. (21) and (22) (**Lyth & Riotto**, 1999):

$$\epsilon_i \equiv \frac{m_P^2}{2}\left(\frac{V_i}{V}\right)^2 , \qquad (23)$$

$$\eta_i \equiv m_P^2\frac{V_{ii}}{V} . \qquad (24)$$

By writing the slow-roll parameters in terms of derivatives of the scalar potential, as in the last two expressions, we realize that the slow-roll conditions require very flat potentials to be met.

Since I am considering a slow-roll regime for the model given by the potential in Eq. (15), the evolution of the fields in such a case is given by

$$\phi(N) = \phi_\star \exp(-N\eta_\phi) , \qquad (25)$$

and

$$\sigma(N) = \sigma_\star \exp(-N\eta_\sigma) , \qquad (26)$$

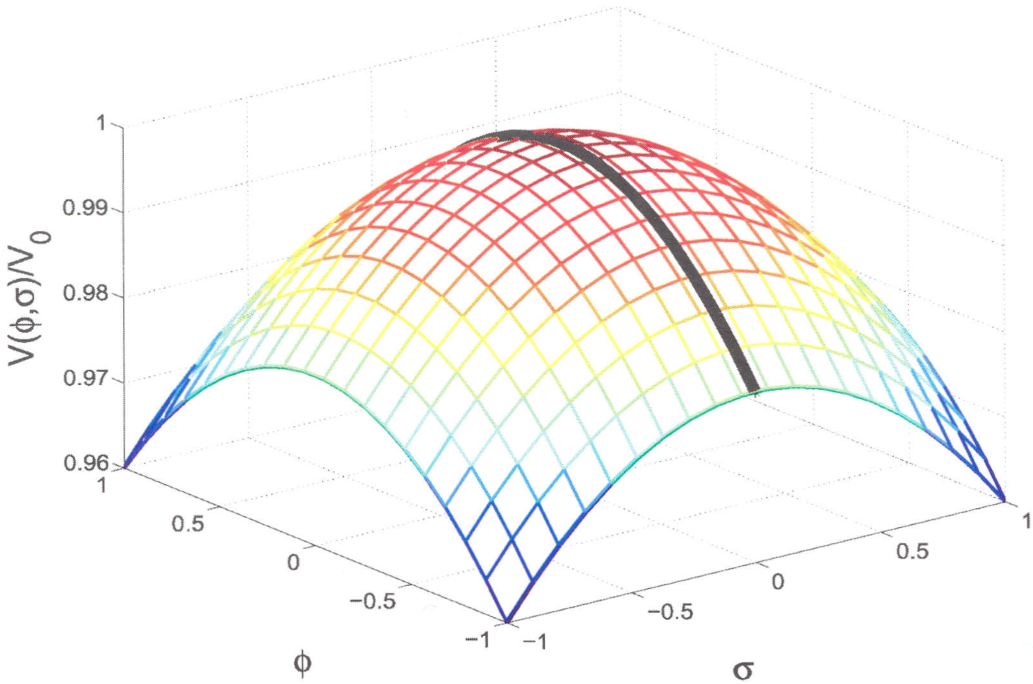

Figure 1: My small-field slow-roll potential of Eq. (15) with $\eta_\phi, \eta_\sigma < 0$. The inflaton starts near the maximum and moves away from the origin following the $\sigma = 0$ trajectory depicted with the solid black line. (This figure has been taken from Ref. (**Alabidi**, 2006)).

in terms of the amount of inflation N since horizon exit, and the field values $\phi_\star$ and $\sigma_\star$ at the time when the relevant cosmological scales exit the horizon. Such expressions, together with Eq. (15), seed the δN formalism in order to calculate the spectrum and the bispectrum of the curvature perturbation including the tree-level (see Fig. 2) and the one-loop contributions (see Fig. 3)[10] (see the respective calculational details in Refs. **Byrnes, Koyama, Sasaki, & Wands**, 2007; **Cogollo, Rodríguez, & Valenzuela-Toledo**, 2008a; **Lyth & Rodríguez**, 2005a; **Sasaki & Stewart**, 1996):

$$\mathcal{P}_\zeta^{tree} = \frac{1}{\eta_\phi^2 \phi_\star^2}\left(\frac{H_\star}{2\pi}\right)^2, \quad (27)$$

$$\mathcal{P}_\zeta^{1-loop} \simeq \frac{\eta_\sigma^2}{\eta_\phi^4 \phi_\star^4} \exp[4N(|\eta_\sigma| - |\eta_\phi|)] \times \left(\frac{H_\star}{2\pi}\right)^4 \ln(kL), \quad (28)$$

$$B_\zeta^{tree} = -\frac{1}{\eta_\phi^3 \phi_\star^4}\left(\frac{H_\star}{2\pi}\right)^4 4\pi^4 \left(\frac{\sum_i k_i^3}{\prod_i k_i^3}\right), \quad (29)$$

$$B_\zeta^{1-loop} \simeq \frac{\eta_\sigma^3}{\eta_\phi^6 \phi_\star^6} \exp[6N(|\eta_\sigma| - |\eta_\phi|)] \times \left(\frac{H_\star}{2\pi}\right)^6 \ln(kL) 4\pi^4 \left(\frac{\sum_i k_i^3}{\prod_i k_i^3}\right), \quad (30)$$

where L is the infrared cutoff chosen so that the quantities are calculated in a minimal box (**Lyth**, 2007), i.e. $\ln(kL) \sim$

[10]For an adequate explanation of the Feynman-like diagrams in cosmology, and their application within the δN formalism, see Ref. (**Byrnes, Koyama, Sasaki, & Wands**, 2007).

Figure 2: Tree-level Feynman-like diagrams for (a) P_ζ, and (b) B_ζ. The internal dashed lines correspond to two-point correlators of field perturbations.

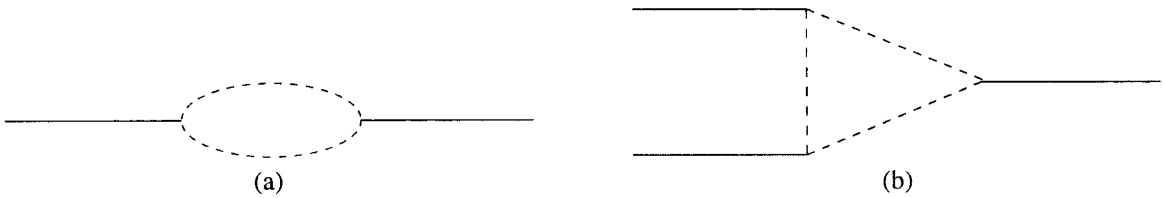

Figure 3: One-loop Feynman-like diagrams for (a) P_ζ, and (b) B_ζ. The internal dashed lines correspond to two-point correlators of field perturbations.

$\mathcal{O}(1)$, and $k_1 \sim k_2 \sim k_3$.

The important factor in the loop corrections is the exponential. This exponential function is directly related to the quadratic form of the potential with a leading constant term. It will give a large contribution if $|\eta_\sigma| > |\eta_\phi|$. I have chosen the concave downward potential in order to satisfy the spectral tilt constraint, which makes either $\eta_\phi < 0$, if $\mathcal{P}_\zeta \simeq \mathcal{P}_\zeta^{tree}$, or $\eta_\sigma < 0$, if $\mathcal{P}_\zeta \simeq \mathcal{P}_\zeta^{1-loop}$, while keeping $|\eta_\sigma| > |\eta_\phi|$.

4 Constraints to have a reliable parameter space

4.1 Convergence of the ζ series and existence of a perturbative regime

It has been proved (**Cogollo, Rodríguez, & Valenzuela-Toledo**, 2008a; **Cogollo, Rodríguez, & Valenzuela-Toledo**, 2008b) by means of a non-perturbative approach, that there exist what are called two "coupling constants" x and y for the potential in Eq. (15). Such coupling constants allow us to obtain a necessary condition for the convergence of the ζ series and work in a perturbive regime if they are much less than one:

$$|x| \equiv \left|\frac{\delta\phi_\star}{\phi_\star}\right| \approx \left(\frac{H_\star}{2\pi}\right)\frac{1}{\phi_\star} \ll 1 \,, \tag{31}$$

$$\begin{aligned} |y| &\equiv \left\{\frac{\eta_\sigma^3}{\eta_\phi^3}\frac{\delta\sigma_\star^2}{\phi_\star^2}\exp[2N(\eta_\phi-\eta_\sigma)]\right\}^{1/2} \\ &\approx \left\{\frac{\eta_\sigma^3}{\eta_\phi^3}\left(\frac{H_\star}{2\pi}\right)^2\frac{1}{\phi_\star^2}\exp[2N(\eta_\phi-\eta_\sigma)]\right\}^{1/2} \ll 1 \,. \end{aligned} \tag{32}$$

4.2 Tree-level or loop dominance

Because of the exponential factors in Eqs. (28) and (30) it might be possible that the loop corrections dominate over $\mathcal{P}_\zeta$ and/or B_ζ. There are three posibilities in complete connection with the position of the ϕ field when the relevant scales are exiting the horizon. Here I will consider only the intermediate $\phi_\star$ region, corresponding to the case when B_ζ is dominated by one-loop corrections and $\mathcal{P}_\zeta$ is dominated by the tree-level term, because this is the only possibility which gives interesting ar.d observationally relevant results.

B_ζ dominated by one-loop corrections and $\mathcal{P}_\zeta$ dominated by the tree-level term: the intermediate $\phi_\star$ region

Looking at Eqs. (27) and (30) I require in this case that

$$\frac{\eta_\sigma^2}{\eta_\phi^2}\exp[4N(|\eta_\sigma|-|\eta_\phi|)] \ll \frac{1}{\frac{1}{\phi_\star^2}\left(\frac{H_\star}{2\pi}\right)^2}, \quad (33)$$

$$\frac{\eta_\sigma^3}{\eta_\phi^3}\exp[6N(|\eta_\sigma|-|\eta_\phi|)] \gg \frac{1}{\frac{1}{\phi_\star^2}\left(\frac{H_\star}{2\pi}\right)^2}, \quad (34)$$

which combine to give

$$\frac{r\mathcal{P}_\zeta}{8}\frac{\eta_\sigma^2}{\eta_\phi^2}\exp[4N(|\eta_\sigma|-|\eta_\phi|)] \ll \left(\frac{\phi_\star}{m_P}\right)^2 \ll \frac{r\mathcal{P}_\zeta}{8}\frac{\eta_\sigma^3}{\eta_\phi^3}\exp[6N(|\eta_\sigma|-|\eta_\phi|)], \quad (35)$$

where the definition for the tensor to scalar ratio r (**Lyth**, 2008) has been employed:

$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_\zeta} = \frac{\frac{8}{m_P^2}\left(\frac{H_\star}{2\pi}\right)^2}{\mathcal{P}_\zeta}. \quad (36)$$

In the latter expression, $\mathcal{P}_T^{1/2}$ represents the amplitude of the spectrum for primordial gravitational waves.

4.3 Spectrum normalisation condition

Since I am considering ζ being generated during inflation, I must satisfy the appropiate spectrum normalisation condition. According to Eq. (27) if $\mathcal{P}_\zeta$ is dominated by the tree-level term, I have

$$\mathcal{P}_\zeta^{tree} = \frac{1}{\eta_\phi^2\phi_\star^2}\left(\frac{H_\star}{2\pi}\right)^2 = \frac{1}{\eta_\phi^2}\left(\frac{m_P}{\phi_\star}\right)^2\frac{r\mathcal{P}_\zeta}{8} = \mathcal{P}_\zeta, \quad (37)$$

which reduces to

$$\left(\frac{\phi_\star}{m_P}\right)^2 = \frac{1}{\eta_\phi^2}\frac{r}{8}. \quad (38)$$

Notice that in such a situation, the value of the ϕ field when the relevant scales are exiting the horizon depends exclusively on the tensor to scalar ratio, once η_ϕ has been fixed by the spectral tilt constraint.

4.4 Spectral tilt constraint

The current observed value for the spectral tilt is $n_\zeta - 1 = -0.040 \pm 0.014$ (**Komatsu et. al.**, 2008), and again I will consider only the case when $\mathcal{P}_\zeta$ is dominated by the tree-level term. That means that the usual spectral index formula (**Sasaki & Stewart**, 1996) applies:

$$n_\zeta - 1 = -2\epsilon - 2m_P^2\frac{\sum_{ij} V_i N_j N_{ij}}{V\sum_i N_i^2}, \quad (39)$$

giving the following result once the derivatives of N with respect to $\phi_\star$ and $\sigma_\star$ have been calculated:

$$n_\zeta - 1 = -2\epsilon + 2\eta_\phi. \quad (40)$$

The efect of the ϵ parameter may be discarded in the previous expression since ϵ is much less than $|\eta_\phi|$:

$$\epsilon = \frac{m_P^2}{2}\frac{V_\phi^2+V_\sigma^2}{V^2} = |\eta_\phi|\left[\frac{1}{2}|\eta_\phi|\left(\frac{\phi}{m_P}\right)^2\right] \ll |\eta_\phi|, \quad (41)$$

according to the prescription that the potential in Eq. (15) is dominated by the constant term. Thus, using the central value for $n_\zeta - 1$, I get

$$\eta_\phi = -0.020. \quad (42)$$

4.5 Amount of inflation

It is well known that the number of e-folds of expansion from the time the cosmological scales exit the horizon to the end of inflation is presumably around but less than 62 (**Dodelson**, 2003; **Liddle & Lyth**, 2000; **Weinberg**, 2008). The slow-roll evolution of the ϕ field in Eq. (25) tells us that such an amount of inflation is given by

$$N = \frac{1}{|\eta_\phi|}\ln\left(\frac{\phi_{end}}{\phi_\star}\right) \lesssim 62, \quad (43)$$

where ϕ_{end} is the value of the ϕ field at the end of inflation. Because of the characteristics of the inflationary potential in Eq. (15), there is no a definite mechanism to end inflation in this model. It could not be by means of the violation of the $\epsilon < 1$ condition since this would imply extrapolating our results to a region where the potential in Eq. (15) is no longer dominated by the constant term which, in addition, would spoil the large non-gaussianity generated and could send the model to an unknowable quantum gravity regime. Keeping in mind the results of Ref. (**Armendariz-Picon, Fontanini, Penco, & Trodden**, 2008) which say that the ultraviolet cutoff in cosmological perturbation theory could be a few orders of magnitude bigger than m_P, I will therefore assume that inflation comes to an end when $|\eta_\phi|\phi^2/2m_P^2 \sim 10^{-2}$. This allows me to be in a safe side (avoiding large modifications to the potential coming from ultraviolet cutoff-suppressed non-renormalisable terms, and keeping the potential dominated by the constant V_0 term), leaving the implementation of a mechanism to end inflation for a future work[11]. Coming back to Eq. (43), I get then

$$N = \frac{1}{|\eta_\phi|} \ln\left(\frac{m_P}{\phi_\star}\right) \lesssim 62\,, \qquad (44)$$

which leads to

$$\frac{\phi_\star}{m_P} \gtrsim \exp(-62|\eta_\phi|)\,. \qquad (45)$$

5 f_{NL}

In this section I will calculate the level of non-gaussianity represented in the parameter f_{NL} (**Komatsu & Spergel**, 2001) by taking into account the constraints presented in Section 4 (**Cogollo, Rodríguez & Valenzuela-Toledo**, 2008a). The level of non-gaussianity, according to the expressions in Eqs. (12), (27), and (30), is in this case given by

$$\begin{aligned} \frac{6}{5} f_{NL} &= \frac{B_\zeta^{1-loop}}{4\pi^4 \frac{\sum_i k_i^3}{\prod_i k_i^3} (\mathcal{P}_\zeta^{tree})^2} \\ &\simeq \frac{\eta_\sigma^3}{\eta_\phi^2 \phi_\star^2} \exp[6N(|\eta_\sigma| - |\eta_\phi|)] \left(\frac{H_\star}{2\pi}\right)^2 \ln(kL) \\ &= \frac{\eta_\sigma^3}{\eta_\phi^2} \exp[6N(|\eta_\sigma| - |\eta_\phi|)] \left(\frac{m_P}{\phi_\star}\right)^2 \frac{r\mathcal{P}_\zeta}{8} \ln(kL) \\ &= \eta_\sigma^3 \exp[6N(|\eta_\sigma| - |\eta_\phi|)] \mathcal{P}_\zeta \ln(kL)\,, \qquad (46) \end{aligned}$$

$$\Rightarrow \frac{6}{5} f_{NL} \approx -2.457 \times 10^{-9} |\eta_\sigma|^3 \exp\Big[300 \times \times \ln(5.657 \times 10^{-2} r^{-1/2}) (|\eta_\sigma| - 0.020)\Big]\,, \qquad (47)$$

where in the last line I have used expressions in Eqs. (38), (42), and (44).

In figure 4 I show lines of constant f_{NL} in the plot r vs $|\eta_\sigma|$ for the intermediate $\phi_\star$ region in agreement with the constraints in Eqs. (31), (32), and (35). Notice that by implementing the spectral tilt constraint in Eq. (42) to the spectrum normalisation constraint in Eq. (38) and the amount of inflation constraint in Eq. (45) I conclude that the tensor to scalar ratio is bounded from below: $r \gtrsim 2.680 \times 10^{-4}$.

6 Conclusions

As is evident from the plot, the WMAP (and also PLANCK) observationally allowed 2σ range of values for negative f_{NL}, $-9 < f_{NL}$, is completely inside the intermediate $\phi_\star$ region as required. More negative values for f_{NL}, up to $f_{NL} = -20.647$ are consistent within my framework for the intermediate $\phi_\star$ region, but they are ruled out from observation. Nevertheless, it is interesting to see a slow-roll inflationary model with canonical kinetic terms where the observational restriction on f_{NL} may be violated *by an excess* and not by a shortfall. So I conclude that *if B_ζ is dominated by the one-loop correction but P_ζ is dominated by the tree-level term, sizeable non-gaussianity is generated even if ζ is generated during inflation.* I also conclude, from looking at the small values that the tensor to scalar ratio r takes in figure 4 compared with the present technological bound $r \gtrsim 10^{-3}$ (**Friedman, Cooray, & Melchiorri**, 2006), that *for non-gaussianity to be observable in this model, primordial gravitational waves must be undetectable.*

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[11] I hope that the implementation of such a mechanism in my model will keep, or perhaps enhance, the generated non-gaussianity. Nevertheless the opposite behaviour might as well happen. For instance, Ref. (**Rigopoulos, Shellard, & van Tent**, 2007) studies within a stochastic formalism a quadratic two-component slow-roll model without a dominant constant term in the potential. A momentary violation of the slow-roll conditions around the end of inflation shows to enhance f_{NL} to observable levels; however, such an enhancement vanishes once inflation ends completely. These results have been confirmed numerically within the δN formalism in Refs. (**Vernizzi & Wands**, 2006; **Yokoyama, Suyama, & Tanaka**, 2008a).

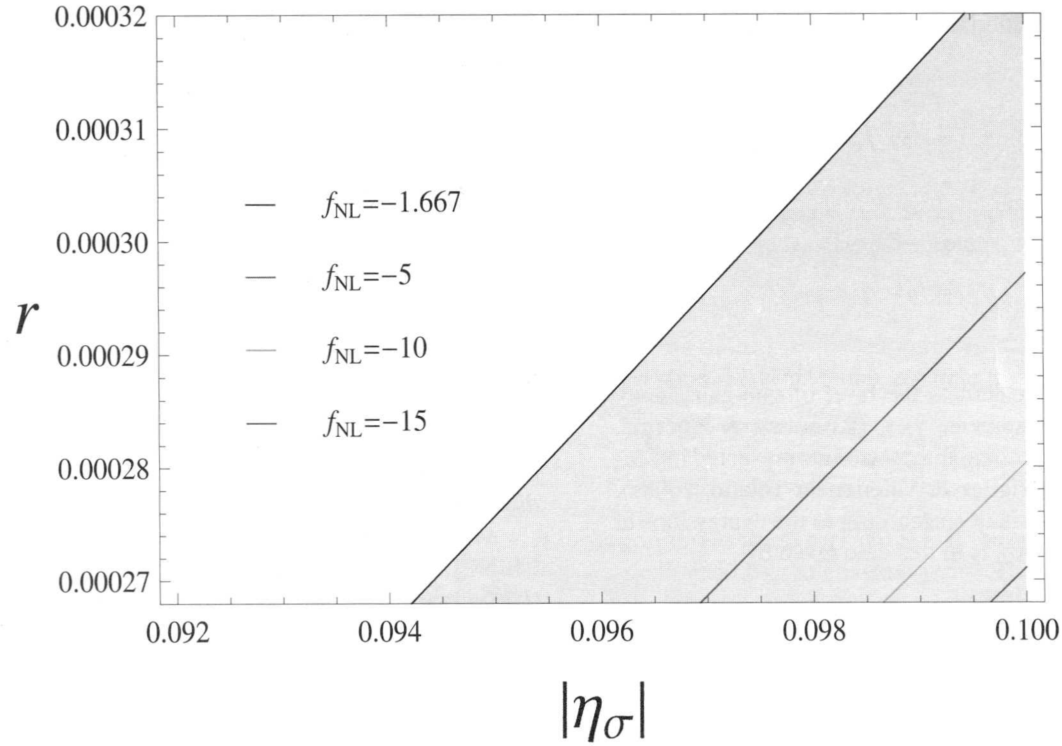

Figure 4: Contours of f_{NL} in the r vs $|\eta_\sigma|$ plot. The intermediate (high) $\phi_\star$ region corresponds to the shaded (white) region. The WMAP (and also PLANCK) observationally allowed 2σ range of values for negative f_{NL}, $-9 < f_{NL}$, is completely inside the intermediate $\phi_\star$ region. Notice that the boundary line between the high and the intermediate $\phi_\star$ regions matches almost exactly the $f_{NL} = -1.667$ line.

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