INSOLATION AT THE EARTH'S SURFACE

by

Joaquín Pelkowski[1]

Abstract

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The (variable) transparency of our atmosphere regulates the amount of solar energy reaching the ground. Also known as clearness index or transmissivity, it has been related linearly to relative sunshine hours ever since the seminal work of Ångström, whose regression model was later modified and generalized, to become a classic tool in the statistical assessment of the global radiation at the surface of the Earth. In this paper we submit a physico-meteorological rationale for the polynomial regression between clearness index and relative sunshine duration.

Key words: Ångström-Prescott regression, solar surface irradiance, radiative transfer.

Resumen

La transparencia (variable) de nuestra atmósfera regula la cantidad de energía solar recibida al nivel del mar. Conocida también como índice de claridad o transmisividad, se ha relacionado linealmente con la heliofanía desde el trabajo influyente de Ångström, cuyo modelo de regresión fue modificado y generalizado más tarde, convirtiéndose en una herramienta clásica para la estimación estadística de la irradiación solar en superficie. En el presente artículo proponemos una base físico-meteorológica para la regresión polinomial entre el índice de claridad y la heliofanía relativa.

Palabras clave: Insolación en superficie, regresión de Ångström-Prescott, transferencia radiactiva.

1 Castillo de Capellomonte, Alemania. Correo electrónico: jo-ellen.pelkowski@t-online.de

1. Introduction

A vast literature seems to have sprung from a single source, an article published in 1924 by **Anders Ångström**, in which a linear relation between a relative short-wave transparency of the atmosphere and a relative sunshine duration is put forward. It was later modified by **Prescott** (1940), who replaced *relative* with *absolute* transparency.

Sunshine recordings are more widely available than irradiation measurements. For that reason, attempts have been and still are made to find, by regressional methods, the coefficients of what is often called the Ångström-Prescott formula.

For many practical purposes, the linear regression has proved valuable. Recently, however, a quadratic dependence on relative sunshine duration has been shown to reproduce measured values better, in a statistical sense.

Alternatively, the coefficients in the linear relation have been taken to be themselves functions of the regressor variable, giving thereby rise to such a quadratic dependency of transparency (or clearness index) on relative (or fractional) sunshine duration (**Akinoglu** 2008). Even a third-order polynomial regression between the two quantities has been used for data quality control purposes (**IDEAM & UPME** 2005).

The urge for physical underpinning of the regression coefficients has been recurrent, and some of the most recent attempts, of an *ad hoc* nature, towards that end have been surveyed by **Akinoglu** (2008). On the other hand, the theory of radiative transfer through atmospheres does afford a physical foundation for the calculation of atmospheric transparency.

Yet it is too elaborate to be of practical use, particularly when only limited measurements are available; little wonder, then, that there is still much room for the kind of simple statistical modeling that has been carried out in the spirit of Ångström's linear regression, a telling example being the new book edited by **Badescu** (2008).

Convinced of the assets of both approaches, a rationale for systematically deriving regression formulae of Ångström-Prescott-type is propounded on the basis of a simple two-stream model of radiative transfer. But first we need to look into the original formulae more closely.

2. Ångström's formula and its generalizations

As already asserted, **Ångström** (1924) proposed a linear relation between a relative transparency of the atmosphere for solar radiation and the relative sunshine duration, to wit:

$$\overline{\mathcal{T}_A} = a_A + b_A\bar{\eta} \tag{1}$$

where

$$\overline{\mathcal{T}_A} \equiv \frac{\overline{Q^{\downarrow}(\tau_b)}}{\overline{Q_0^{\downarrow}(\tau_b)}} \tag{2}$$

defines a certain (mean) transparency and

$$\bar{\eta} \equiv \frac{\bar{h}}{H} \tag{3}$$

is often referred to as the (average) relative (or fractional) sunshine duration. $\overline{Q^{\downarrow}}(\tau_b)$ we define here as the mean flux density (irradiance, insolation) at the (horizontal) surface of the Earth; likewise, $\overline{Q_0^{\downarrow}}(\tau_b)$ is the mean flux density, but for a sunny, cloudless day. τ_b is the optical depth of the whole atmosphere, a measure of its opacity with respect to solar radiation; $\bar{h}$ is represents a convenient mean of h, itself being the number of sunshine hours during a whole day, while H stands for the maximum possible amount of sunshine at a certain geographical place and time of the year. It can be calculated from astronomical considerations of the insolation distribution on the top of the Earth's atmosphere.

The bars appearing in these equations remind us that Ångström rightly saw his formula to be applicable only to climatological values of transparency and sunshine, not to instantaneous ones. The period over which has to be averaged is a matter of choice; it could refer to monthly values, by averaging the daily recordings of solar radiation and sunshine; or it could represent seasonal averages, even yearly or decadal ones. Furthermore, all quantities involved are, in principle, spectral ones, and therefore they depend on the wavelength of the solar radiation reaching sea level. We shall omit both the bars and any reference to the wavelength dependency, on the understanding that the quantities involved in this article are monochromatic averages, unless otherwise stated. As we will not be dealing with instantaneous irradiation, the model to be laid down disregards from the outset the actual elevation of the sun's disk as it varies with the hour of a day, and thus delivers at best daily averages of insolation.

Formula (1) was modified by **Prescott** (1940) to read

$$\mathcal{T} \equiv \frac{Q^{\downarrow}(\tau_b)}{Q^{\downarrow}(0)} = \mathrm{a} + \mathrm{b}\eta, \tag{4}$$

who introduced absolute transparency, $Q^{\downarrow}(0)$ now standing for the solar flux density at the top of the atmosphere (where optical depth vanishes). Also known as insolation, it is expressed by the formula

$$Q^{\downarrow}(0) = \left(\frac{D_0}{D}\right)^2 Q_0 \cos\theta_0, \tag{5}$$

in which Q_0 is the (monochromatic) extraterrestrial normal solar flux density (known as the solar constant), θ_0 being the sun's (disk) zenith angle, measured with respect to the local normal. D denotes the actual distance of the Earth from the Sun, D_0 being an average distance (the astronomical unit). The daily average of this expression follows upon integrating (5) over a day's period, with the understanding that $Q^{\downarrow}(0)$ vanishes during the night ($\theta_0 > \pi/2$):

$$\left(\frac{D_0}{D}\right)^2 \frac{Q_0}{\pi}(\Lambda_0 \sin\Phi \sin\delta + \sin\Lambda_0 \cos\Phi \cos\delta).$$

Here, $\pm\,\Lambda_0$ is the hour angle at sunset (+) or sunrise (−) for latitude Φ and declination angle δ. The maximum possible sunshine hours at a certain location, H, used in defining η, and the hour angle Λ_0 (in degrees) are related by $H = 2\Lambda_0/15$. All these definitions are well known from studies of insolation distribution on planets and need not be discussed here any further.

The Ångström-Prescott and the original Ångström formulas may readily be shown to be related as follows:

$$\mathcal{T}_A(\eta) = \frac{\mathcal{T}(\eta)}{\mathcal{T}(1)} \tag{6}$$

and hence $\mathcal{T}_A(1) = a_A + b_A = 1$. We expect measurements of $\mathcal{T}_A(\eta)$ and $\mathcal{T}(\eta)$ to be perfectly correlated. We therefore are not surprised to learn that **Ianetz & Kudish** (2008), who think $\mathcal{T}_A(\eta)$ to be a "better indicator of the degree of cloudiness than the oft used $\mathcal{T}(\eta)$", find them to be highly correlated. They conclude from a statistical analysis of measurements at Beer Sheva that the regression coefficients for monthly averages of both transparencies explain almost 100% of the data variance.

Many values for the Ångström-Prescott coefficients a and b have been published for different places; it is not the purpose of this article to review them. For the sake of illustration, I quote only a pair purported to be representative of the whole of France: a = 0.2 and b = 0.55 (**Guyot** 1998). But we must keep in mind that the values do vary from place to place, depending also on the averaging period. According to **Akinoglu** (2008), they range between the following limits: $0.06 \leq \mathrm{a} \leq 0.46$ and $0.19 \leq \mathrm{b} \leq 0.87$.

We pointed out above that efforts to generalize the formula of Ångström or Ångström-Prescott are not uncommon. It seems that a quadratic relationship between $\mathcal{T}$ and η produces better results when compared with some data (**Akinoglu** 2008), and even higher order polynomials have been proposed (**Şahin & Şen** 2008). A third-order polynomial was deemed necessary by the authors of the solar radiation atlas of Colombia (**IDEAM & UPME** 2005), from where we quote, for later reference, the following numerical example (correlation coefficient of 0.56)[2]:

$$\mathcal{T} = 0.343 + 0.244\eta + 0.113\eta^2 - 0.026\eta^3. \tag{7}$$

Whatever the specific reasons for such generalizations, it does seem clear that expanding Ångström's original regression model to allow for a polynomial regression between $\mathcal{T}$ and η is a natural step. If the regression coefficients could be shown to derive from a physical model of the relation between insolation and sunshine, our resolve to carry out all the statistical inferences necessary to validate the regression models would thereby be strengthened .

With this question of the physical origin of statistical coefficients in mind it is that we put forward a physico-meteorological rationale for the polynomial regression between the transparency of the atmosphere and the relative sunshine duration.

3. A model atmosphere

A model atmosphere simple enough will allow us to achieve our aim set forth in the last paragraph. Two-stream models have been found to be parsimonious approximations to the radiative transfer of electromagnetic energy through a plane-parallel atmosphere. In their broadband or semi-gray versions, they enable one to calculate the transmissivity (transparency) of the model atmosphere, either with respect to solar or thermal radiation. I shall not discuss two-stream models here, of which differing versions can be found in many textbooks.

[2]Unfortunately, the atlas is silent as to locality and period for which the values apply, so this equation should be taken as a mere numerical illustration. I regret not being able to comply with the request of a reviewer, who wished to know the meteorological circumstances surrounding the regression, so as to bear out our interpretation, in Sec.3.2.1, of the negative coefficient.

While I shall be using my own version (**Pelkowski** 2007), any other model of the same kind may be laid down. For most details, however, the reader may also be referred to the textbook of **Petty** (2006), or to the lucid and briefer exposition by **Bohren** (1987).

The transparency of a plane-parallel atmosphere–a good model for the vertical energy fluxes in the real atmosphere–can be calculated starting from the equation of (monochromatic) radiative transfer, which we write as:

$$\mu \frac{\partial I(\tau,\mu)}{\partial \tau} = I - \frac{a}{2}\int_{-1}^{1} P(\mu,\mu')I(\tau,\mu')d\mu'. \quad (8)$$

The symbols have the usual meanings[3]:

I: radiance (intensity) of a light ray inclined with respect to the local normal by a zenith angle $\theta = \cos^{-1}\mu$

τ: optical depth into the atmosphere at altitude z,

$$\tau(z) = \int_{z}^{H_t} k\rho dz', \quad (9)$$

in which k is the (specific or mass) extinction coefficient, ρ the density of the optically active matter, and H_t the height of the top of the atmosphere

P: phase function, describing the angular distribution of radiance at a point where the incident radiation with zenith angle $\theta' = \cos^{-1}\mu'$ is being scattered into the ray with zenith angle $\cos^{-1}\mu$.

a: scattering albedo (the ratio of scattering coefficient to total extinction coefficient).

Note that in general we should include a thermal source of photons emitted by matter at the local temperature, a source that however barely contributes to the shortwave energy *fluxes*. It can be safely neglected in our context.

By next introducing the downward and upward shortwave energy flux densities at any depth τ, $Q^{\downarrow}(\tau)$ and $Q^{\uparrow}(\tau)$, respectively, defined as

$$Q^{\downarrow}(\tau) = -2\pi\int_{-1}^{0} I(\tau,\mu)\mu d\mu > 0$$

and

$$Q^{\uparrow}(\tau) = 2\pi\int_{0}^{+1} I(\tau,\mu)\mu d\mu > 0,$$

we may set up equations apiece, starting from the radiative transfer equation (8) (**Petty** 2006, **Pelkowski** 2007):

$$\mu^{+}\frac{dQ^{\downarrow}(\tau)}{d\tau} = -(1-af)Q^{\downarrow}(\tau) + a(1-f)Q^{\uparrow}(\tau) \quad (10)$$

and

$$\mu^{+}\frac{dQ^{\uparrow}(\tau)}{d\tau} = (1-af)Q^{\uparrow}(\tau) - a(1-f)Q^{\downarrow}(\tau). \quad (11)$$

To derive these equations, some simplifying assumptions were made, an important one being the choice of the phase function. $\mu^{+} > 0$ is independent of τ, but it may vary with wavelength and total optical depth τ_b, defined by Eq. (9) as $\tau_b = \tau(0)$, as shown in **Pelkowski** (2007). Note that $\mu^{+} \in (0,1]$. In **Petty**'s (2006) two-stream equations, $\mu^{+} = 0.5$, while **Bohren**'s (1987) two-stream version corresponds to $\mu^{+} = 1$. In our model, a singular phase function was chosen so that a fraction f of the energy is effectively scattered into the hemisphere (with equatorial plane parallel to the atmospheric layers) into which the energy flows, the complementary fraction $1-f$ being back-scattered. Fraction f is shown in my previous account to be related to the asymmetry factor g by $f = (1+g)/2$, while **Petty** (2006) *assumes* it as plausible. The asymmetry factor is a widely used characteristic of the asymmetry in the distribution of scattered radiance, reflecting, to a certain extent, the size, shape and nature of the scattering particles. Plenty of values can be found in the relevant literature.

Eqs. (10) and (11) are ordinary differential equations for the flux densities of solar radiation that cross the atmosphere either downwards or upwards. The value of the downwelling flux density at the surface (where $\tau = \tau_b$) is known as the "global" irradiance. This surface insolation $Q^{\downarrow}(\tau_b)$ comprises both the direct and the diffuse irradiance, which may be kept separate, as is commonly done in studies of its measurement and calculation:

$$Q^{\downarrow}(\tau_b) = Q^{\downarrow}_{\odot}(\tau_b) + Q^{\downarrow}_{*}(\tau_b). \quad (12)$$

However, we shall not need this distinction beforehand, since our model, being applicable at best to daily averages, will not include the daily variation of the hour angle. Only an effective cosine μ^{+} is taken into account, which may be interpreted as an average of the cosine of the sun's zenith angle.

As a side remark, and for later reference, I point out that when no diffuse fluxes are separately measured, only "global" ones, many an investigator has tried to infer the former from the latter. For example, **Boland**

[3]Almost all quantities are spectral ones but we need not remind ourselves every time of this fact.

& Ridley (2008) show that by defining the diffuse *fraction*

$$\mathcal{D} = \frac{Q_*^{\downarrow}(\tau_b)}{Q^{\downarrow}(\tau_b)}, \tag{13}$$

which in our notation we write as

$$\mathcal{D}(\mathcal{T}) = 1 - \frac{\mathcal{T}_{\odot}}{\mathcal{T}}, \tag{14}$$

a logistic function can be fitted to the data between $\mathcal{D}$ and $\mathcal{T}$. $\mathcal{D}$ tends to be small when the transmissivity is dominated by the direct component $\mathcal{T}_{\odot} = Q_{\odot}^{\downarrow}(\tau_b)/Q^{\downarrow}(0)$, but if the direct component is absent (as in overcast skies), then $\mathcal{D} = 1$.

The transparency of the atmosphere, $\mathcal{T} = Q^{\downarrow}(\tau_b)/Q^{\downarrow}(0)$, can now be determined for an optically uniform model atmosphere (for which a and g are independent of position within the layer) from the solution to Eq. (10) under appropriate boundary conditions (**Petty** 2006), including the solar energy reflected isotropically at the surface, whenever its albedo r_s does not vanish. The following expression for the transparency is thus arrived at (**Pelkowski** 2007):

$$\mathcal{T} = \frac{1 - r_{\infty}^2}{e^{\gamma\tau_b^*} - r_{\infty}^2 e^{-\gamma\tau_b^*} - r_s r_{\infty}(e^{\gamma\tau_b^*} - e^{-\gamma\tau_b^*})}, \tag{15}$$

with abbreviations defined as: $\tau_b^* = \tau_b/\mu^+$,

$$\gamma \equiv \sqrt{(1-a)(1-ag)} \tag{16}$$

and

$$r_{\infty} = \frac{\sqrt{1-ag} - \sqrt{1-a}}{\sqrt{1-ag} + \sqrt{1-a}}. \tag{17}$$

r_{∞} is the (intrinsic) reflectivity of a semi-infinite scattering layer (see Eq. (20) below, for $\tau_b^* \longrightarrow \infty$).

The transparency may be rewritten more compactly as

$$\mathcal{T} = \frac{t}{1 - r_s r}, \tag{18}$$

by means of the intrinsic transmissivity

$$t = \frac{1 - r_{\infty}^2}{e^{\gamma\tau_b^*} - r_{\infty}^2 e^{-\gamma\tau_b^*}} \tag{19}$$

and the intrinsic reflectivity

$$r = r_{\infty} \frac{1 - e^{-2\gamma\tau_b^*}}{1 - r_{\infty}^2 e^{-2\gamma\tau_b^*}} \tag{20}$$

of the optically uniform atmosphere.

In establishing these quantites, the atmosphere was assumed to be optically uniform, so that the solution of the flux equations (10) and (11) could be found by standard methods. As we wish to take into account the observed vertical variations of optical parameters of the atmosphere, we refine the model by dividing the atmosphere into a minimum number of optically uniform layers. If we agree that cloudiness should be included in a model purporting to relate transparency and daily sunshine duration, we perceive at once that a minimum of three layers is required (**Pelkowski** 2007): a cloud layer with variable cloudiness, a cloudless upper layer and a cloudless lower layer. Every layer is supposed to be optically uniform, and we shall distinguish corresponding quantities in each layer with different subscripts: c refers to cloud properties in the cloud's layer (itself distinguished by the subscript n), Δ refers to the atmosphere free of clouds within that cloud layer (in which the clouds have optical thickness τ_{cb}, the interstices being less deep, $\tau_{\Delta b} = \tau_2 - \tau_1$); u shall refer to the upper layer (of total optical depth $\tau_{ub} = \tau_1$); finally, l is reserved for the lower atmospheric layer (of optical thickness $\tau_{lb} = \tau_b - \tau_2$). All three layers will have a transmissivity and reflectivity described by the above formulae, but with surface albedo being replaced by appropriate albedos, corresponding to the layers underneath (and to which the surface albedo also contributes). Of course, in the middle layer (hosting the clouds) we will have to arrange for the different transmissivities and reflectivities of the clouds and interstices, and a careful accounting of all the solar fluxes bouncing back and forth between the surface and the cloud layer does indeed lead to simple weighting with cloud amount n:

$$\mathcal{T}_n = \frac{t_n}{1 - A_l r_n} \tag{21}$$

$$t_n = n t_c + (1-n) t_{\Delta} \tag{22}$$

$$r_n = n r_c + (1-n) r_{\Delta}. \tag{23}$$

In Eq. (21), we replaced r_s by the albedo A_l of the system made up by the surface and the lower atmosphere, which would be the albedo of the Earth if the middle and upper layers were not present:

$$A_l = r_l + \frac{r_s t_l^2}{1 - r_s r_l}. \tag{24}$$

This expression may be derived in much the same way as that for the transparency of an optically uniform layer, Eq.(18), by first solving the flux equation for the upward solar radiation, $Q^{\uparrow}(\tau)$, Eq. (11), and then appealing to the definition of albedo:

$$A_l = \frac{Q^{\uparrow}(\tau_2)}{Q^{\downarrow}(\tau_2)}.$$

(For a rigorous approach, see **Chevallier et al.** 2007, or **Kokhanovsky** 2006). The albedo of the upper layer

A_u coincides with the albedo $\mathcal{A}$ of the Earth and is given by the well-known ratio:

$$\mathcal{A} = A_u = \frac{Q^{\uparrow}(0)}{Q^{\downarrow}(0)} = r_u + \frac{A_n t_u^2}{1 - A_n r_u}, \quad (25)$$

($\mathcal{A} = 0.3$ being the presently accepted climatological value); likewise,

$$A_n = \frac{Q^{\uparrow}(\tau_1)}{Q^{\downarrow}(\tau_1)} = r_n + \frac{A_l t_n^2}{1 - A_l r_n} \quad (26)$$

is the albedo of the earth/atmosphere below the depth τ_1 (the top boundary of the middle layer).

We need all these expressions because they appear in our model atmosphere's transparency, which is found by multiplying the transmissivities of the three layers (**Pelkowski** 2007):

$$\mathcal{T} = \mathcal{T}_u \mathcal{T}_n \mathcal{T}_l = \frac{t_u t_n t_l}{(1 - A_n r_u)(1 - A_l r_n)(1 - r_s r_l)}. \quad (27)$$

This is the key expression for the following development. For a discussion of a similar procedure, but from a rather advanced level, the patient reader may wish to consult **Kokhanovsky** (2006).

3.1. **Transparency of the tri-layered model atmosphere.** The transparency (27) may be rewritten as follows:

$$\mathcal{T} = \frac{t}{1 - \varrho} \quad (28)$$

with $t = t_u t_n t_l$ and

$$\varrho = (1 - r_s r_l)[r_n(r_u + A_l) - r_u A_l(r_n^2 - t_n^2)] + r_s r_l. \quad (29)$$

If for a moment we assume that the surface albedo vanishes, together with the optical depths of the middle layer ($\tau_{cb} = \tau_{\Delta b} = 0$, so that $\tau_1 = \tau_2$), this expression reduces to $\varrho = r_u r_l$ and the transmissivity of a two-layer atmosphere is seen to be

$$\mathcal{T} = \frac{t_u t_l}{1 - r_u r_l}, \quad (30)$$

a result that may be found in the literature (**Petty** 2006, **Kokhanovsky** 2006).

Returning to the general expression (28), we remark that since $\varrho < 1$, we may write the transparency as a power series

$$\mathcal{T} = t \sum_{k=0}^{\infty} \varrho^k = t\,(1 + \varrho + \varrho^2 + ...). \quad (31)$$

If it were generally true that $\varrho \ll 1$, we could approximate the transparency by the simpler expression:

$$\mathcal{T} \approx t\,(1 + \varrho). \quad (32)$$

Before proceeding any further, we ask how to relate the cloud fraction n and the fraction of sunshine duration, which we have denoted by η. Can we express the cloudiness as a function of η, $n = f(\eta)$? Certainly a relation between them must exist, though it may not be straightforward to find the right one. If the sun shone for a whole day, i.e., if $\eta = 1$, the day's average of cloudiness may be reasonably thought of as being nil: $f(1) = 0$; vice versa, if no sunshine during the whole day is recorded ($\eta = 0$), we may take that to be due to day-long overcast conditions: $f(0) = 1$. These two extreme values may be connected in a linear or nonlinear way, but we might certainly wish to begin with the simpler linear relation $n = 1 - \eta$, a relation that seemingly has been found to be useful in the literature (cf., e.g., **Robinson** 1966, and **Kondratyev** 1969). Of course, a more general relationship could be envisaged, particularly when cloudiness (as when mountains are capped), not affecting sunshine duration at a site, need somehow to be taken into account. Notwithstanding such a possibility, we shall abide by the linear relation between an average cloudiness and the fractional sunshine duration.

With this relation for n, it is now easy to express the transparency $\mathcal{T}$ of the atmosphere as a function of sunshine duration η. The approximate expression (32), in which both factors depend on η, will suffice for our purposes. By carrying out the multiplication of the two factors, we may arrange the result to produce a third-order polynomial in η:

$$\mathcal{T}(\eta) = a_0 + a_1\eta + a_2\eta^2 + a_3\eta^3 \quad (33)$$

wherein the coefficients are derived functions of reflectivities, albedos and transmissivities, given by the following expressions:

$$\begin{aligned} a_0 &= \mathrm{a}(1 + r_s r_l + \tilde{\gamma}_0) && (34)\\ a_1 &= \mathrm{a}\tilde{\gamma}_1 + \mathrm{b}(1 + r_s r_l + \tilde{\gamma}_0) && (35)\\ a_2 &= \mathrm{b}\tilde{\gamma}_1 - \mathrm{a}\tilde{\gamma}_2 && (36)\\ a_3 &= -\mathrm{b}\tilde{\gamma}_2 && (37) \end{aligned}$$

Here we have introduced the following abbreviations:

$$a = t_u t_c t_l \quad (38)$$

$$b = t_u(t_\Delta - t_c)t_l \quad (39)$$

$$\gamma_0 = r_c(r_u + A_l) - r_u A_l(r_c^2 - t_c^2)$$

$$\gamma_1 = (r_\Delta - r_c)(r_u + A_l) - 2r_u A_l[r_c(r_\Delta - r_c) - t_c(t_\Delta - t_c)]$$

$$\gamma_2 = r_u A_l[(r_\Delta - r_c)^2 - (t_\Delta - t_c)^2]$$

$$\tilde{\gamma}_i = (1 - r_s r_l)\gamma_i \quad i = 0, 1, 2$$

Of course, if ϱ is not small enough, instead of (33) we would get from (31) a polynomial of a higher degree. Indeed, (31) shows that the transparency is a polynomial of infinite degree in η, but we could still get a finite polynomial by lumping together the contributions of certain higher orders in ϱ into a stochastic process, such that

$$\mathcal{T}(\eta) = \sum_{i=0}^{N} a_i \eta^i + \varepsilon.$$

It is clear that the coefficients a_i are in practice random variables, because the meteorological quantities determining them fluctuate more or less randomly. Henceforth we shall restrict our further development to $N = 3$ and shall only consider particular cases of what may be called the "deterministic" transparency (i.e. with $\varepsilon = 0$).

3.2. **Conditional transparencies of the tri-layered atmosphere.**

By examining different special cases, we may simplify the interpretation of the coefficients in Eq. (33). The present section is devoted to some restrictions that lead to simpler forms of those coefficients.

3.2.1. *Non-scattering interstices between clouds.* This case will allow us to infer something about the coefficient a_3, given by (37). When the air in the interstices between the clouds of the middle layer does not scatter solar radiation, $a_\Delta = 0$ and hence $r_\Delta = 0$. It is commonly assumed that clouds are almost conservative scattering media; we may take them to be perfectly so, with the implication that their scattering albedo is unity ($a_c = 1$) and $r_c + t_c = 1$. Our middle layer thus consists of conservative clouds (a good assumption about real clouds) and interstices between them not scattering at all (not an appropriate assumption about real air but one that is not too critical here). Then (37) may, after a few transformations, be written as $a_3 = -t_u(t_\Delta - t_c)t_l(1 - r_s r_l) r_u A_l (1 - t_\Delta)(t_\Delta + 2r_c - 1)$. With typical values for the quantities involved, it is clear that $a_3 < 0$. If the interstices are even totally transparent, i.e. $t_\Delta = 1$, then $a_3 = 0$. In normal conditions t_Δ will not be much less than unity, and so a_3 can be expected to have a small magnitude. Eq. (7) reflects these inferences ($a_3 = -0.026$).

3.2.2. *Non-scattering upper layer.* In this case, the scattering albedo of the upper atmosphere vanishes ($a_u = 0$) and therefore $r_u = 0$, $\gamma_0 = r_c A_l$, $\gamma_1 = (r_\Delta - r_c)A_l$, $\gamma_2 = 0$ and hence $a_3 = 0$. The transparency then reads:

$$\mathcal{T}(\eta) = a_0 + a_1\eta + a_2\eta^2, \tag{40}$$

with accordingly modified coefficients (34), (35), and (36).

3.2.3. *Non-scattering upper and lower layers.* A similar quadratic equation is obtained if both the upper and lower layers do not scatter solar radiation ($a_u = a_l = 0$, implying $r_u = r_l = 0$). Then, from Eqs. (24) and (29) $\varrho = r_n r_s t_l^2$; if the atmosphere consisted of only the middle layer, i.e. the optical depths of the upper and lower layers were nil or negligible, then by Eq. (19) $t_l = 1$ and the original expression for the transparency (27) would read $\mathcal{T} = t_n/(1 - r_s r_n)$, which can be further simplified if we consider only clear skies ($n = 0$):

$$\mathcal{T} = \frac{t_\Delta}{1 - r_s r_\Delta}.$$

Such a global irradiance under cloudless skies is considered by **Kambezidis & Psiloglou** (2008), who call r_Δ the albedo of the cloudless sky after introducing, however, the denominator in an *ad hoc* manner: "...the effect of multiple ground-atmosphere reflections can be accounted for, scaling $[Q^\downarrow(\tau_b)]$ by the adequate factor $(1 - r_s r_\Delta)^{-1}$" (**Kambezidis & Psiloglou** 2008).

3.2.4. *Black surface.* In this case, the surface albedo is zero: $r_s = 0$. Consequently, $\tilde{\gamma}_i = \gamma_i$, and by (24) $A_l = r_l$. The polynomial retains its third-order degree. But if we add the further assumption that the upper and lower layers do not scatter solar radiation (i.e. have vanishing scattering albedos), then $\tilde{\gamma}_i = A_l = 0$ and we get for the transparency the simple linear relationship

$$\mathcal{T}(\eta) = \mathrm{a} + \mathrm{b}\eta \tag{41}$$

in which the "Ångström-Prescott coefficients" become

$$a_0 = \mathrm{a} = t_u t_c t_l \tag{42}$$

$$a_1 = \mathrm{b} = t_u(t_\Delta - t_c)t_l. \tag{43}$$

It goes without saying that a layer that does not scatter solar energy can only transmit it as direct, not diffuse, radiation. Therefore

$$t_x = t_x^\odot = e^{-\tau_{xb}/\mu_x^+}, \qquad x \in \{u, l\} \tag{44}$$

by Eqs.(16), (17) and (19). The superscript symbolizing the sun is to remind us that only direct solar radiation is being dealt with. The diffuse component, t^* may generally be defined as $t - t^\odot$. As we did not allow for instantaneous values, the direct transmissivities are only averages, defined for appropriate "effective" air masses $1/\mu_x^+$, in agreement with the averaging periods laid down.

The transparency for overcast conditions is then, from (41), (42) and (44),

$$\mathcal{T}(0) = \mathrm{a} = t_u^{\odot} t_c t_l^{\odot}. \tag{45}$$

In this limiting case, the transparency is just the product of the direct transmissivities $t_u^{\odot}$, $t_l^{\odot}$ and the total transmissivity t_c of the cloud layer. Direct transmissivities are controlled by the absorption of ozone, water vapor, carbon dioxide, as well as other gases of minor importance; the transmissivity of the cloud layer is affected largely by scattering within the cloud layer, brought about by the air molecules and the cloud elements, and to a lesser degree by absorption of solar energy in water vapor and radiatively active gases, and also by the water itself, albeit this is normally held to be negligible.

Let us turn to the other limiting condition, that of a sunny, cloudless atmosphere. Then (41), together with Eqs. (42), (43) and (44) yield

$$\mathcal{T}(1) = \mathrm{a} + \mathrm{b} = t_u^{\odot} t_c t_l^{\odot} + t_u^{\odot}(t_\Delta - t_c) t_l^{\odot} = t_u^{\odot} t_\Delta t_l^{\odot} \tag{46}$$

which makes perfectly sense, because now we get the transparency of the atmosphere by multiplying all three (cloudless) transmissivities together, that of the middle atmosphere being the (sum of direct and diffuse) transmissivity of the air in the absence of clouds (the "intersticial" transmissivity t_Δ).

Note that from Eqs. (1), (6) and (41), $a_A = \mathrm{a}/(\mathrm{a}+\mathrm{b})$, $b_A = \mathrm{b}/(\mathrm{a}+\mathrm{b})$, and thus a_A is seen to be the ratio of the transparency of the overcast atmosphere to that of the cloudless one, in conformity with Ångström's original meaning.

It should be clear from the preceding account that the classic linear relationship between clearness index (transparency) and relative sunshine duration, known as the Ångström-Prescott formula, cannot, on our grounds, be expected to be valid unless the surface albedo is low and the optical thickness of the cloudless parts of the atmosphere due to (non-conservative) scattering is negligible.

3.2.5. *Cloudless and non-scattering model atmosphere.* In this situation we have: $n = 0$ (i.e. $\eta = 1$); $a_u = a_\Delta = a_l = 0$. Therefore, $r_u = r_\Delta = r_l = 0$ and, choosing the same μ^+ for every layer (cf. also Eq.(51) below),

$$\begin{aligned}\mathcal{T}(1) &= t_u^{\odot} t_\Delta^{\odot} t_l^{\odot} = e^{-\tau_1/\mu^+} e^{-\tau_2/\mu^+} e^{-(\tau_b - \tau_1 - \tau_2)/\mu^+} \\ &= e^{-\tau_b/\mu^+}.\end{aligned} \tag{47}$$

A possible choice for the effective air mass $1/\mu^+$ is $-\log[2E_3(\tau_b)]/\tau_b$ (**Pelkowski** 2007), for then the transparency would correspond to the annual global mean of the direct transmissivity, with the exponential function being replaced by twice the exponential integral $E_3(\tau_b)$, which is just what one gets for the so-called direct spherical transmissivity (**Chevallier et al.** 2007, **Pelkowski** 2007). There are other possibilities to define an effective air mass which we need not discuss here.

If total optical depth is expressible as the sum of the optical depths due to ozone, water vapor, well-mixed absorbing gases and aerosol absorption, then $\tau_b = \tau_{b,O_3} + \tau_{b,H_2O} + \tau_{b,\mathrm{mg}} + \tau_{b,\mathrm{aer}}$ and we could write the transparency of the atmosphere as

$$\mathcal{T}(1) = e^{-\tau_b/\mu^+} = \mathcal{T}_{O_3} \mathcal{T}_{H_2O} \mathcal{T}_{\mathrm{mg}} \mathcal{T}_{\mathrm{aer}},$$

which is what one often finds in studies of (direct or beam) solar radiation at the surface of the Earth (cf., e.g., **Kambezidis & Psiloglou** 2008).

3.2.6. *Conservative cloud layer with overcast skies over black surface.* This situation is described by $r_c + t_c = 1$, $n = 1$ and $r_s = 0$. Then, from (33), (34) and (38), together with (24),

$$\begin{aligned}\mathcal{T}(0) &= a_0 \\ &= t_u(1 - r_c) t_l [1 + r_u r_l + r_c(r_u + r_l - 2 r_u r_l)].\end{aligned} \tag{48}$$

If in addition the cloudless air would not scatter solar radiation, this expression would reduce to Eq. (45). It is well known that for a conservative cloud the transmissivity is given by

$$t_c = \frac{\mu_c^+}{\mu_c^+ + (1 - f_c)\tau_{cb}}, \tag{49}$$

τ_{cb} being the optical depth of the cloud layer. **Petty** (2006) derives (49) for $\mu_c^+ = 0.5$, but it has often been presented for $\mu_c^+ = 1$. Some flexibility is gained by allowing μ_c^+ to assume any value between 0 and 1, but how should we choose it? A criterion discussed in my previous work (**Pelkowski** 2007) is to demand that this cloud transmissivity reproduce the rigorous value in the case of isotropic scattering, which corresponds to $f_c = 0.5$, for then there exist exact expressions (cf., e.g., **Rutily et al.** 2008). For the sake of illustration, suffice it to quote the value $\mu_c^+ = 0.663$ for a cloud of optical depth $\tau_{cb} = 16$ (**Pelkowski** 2007). Of course, one may stick to a single constant value for μ_c^+ if one wishes to eschew the onerous review of the formulae involved. A typical value found in the literature is $\mu_c^+ = 1/\sqrt{3}$ (the "diffusivity factor").

For the transparency (45) we can now write $\mathcal{T}(0) = t_u^{\odot} t_l^{\odot}/[1+(1-f_c)\tau_{cb}/\mu^+]$. It is diminished by the presence of the cloud layer (unless $f_c = 1$, which case is tantamount to no cloud at all). If the cloud elements would strongly backscatter ($f_c \ll 1$)–which they do not–, the transparency of the atmosphere would fall more than for the case of isotropic or Rayleigh scattering ($f_c = 0.5$).

3.2.7. *The general limit transparencies.* In the general case for which we obtained the polynomial equation (33), we may write for the limiting transparencies the following equations:

$$\mathcal{T}(0) = a_0 = t_u t_c t_l [1 + r_s r_l + (1 - r_s r_l)\Gamma_c] \tag{50}$$

$$\begin{aligned}\mathcal{T}(1) &= a_0 + a_1 + a_2 + a_3 \\ &= t_u t_\Delta t_l [1 + r_s r_l + (1 - r_s r_l)\Gamma_\Delta]\end{aligned} \tag{51}$$

where

$$\Gamma_x = r_x(r_u + A_l) - r_u A_l (r_x^2 - t_x^2) \quad x = c \,\text{or}\, \Delta \tag{52}$$

In periods of overcast skies, the transparency of the model atmosphere is enhanced over a reflecting surface; likewise, for cloudless conditions we discern that it also increases over places whose surfaces are not black. I shall illustrate this in the next subsection, devoted to the diffuse transmissivity of one of our specialized model atmospheres.

3.2.8. *Diffuse transmissivities under cloudy skies.* By subtracting the direct transmissivity from the transparency (or total transmissivity) given by (33), we get the diffuse transmissivity $\mathcal{T}_*(\eta) = Q_*^{\downarrow}(\tau_b)/Q^{\downarrow}(0)$. By (14) it may be written in terms of the diffuse fraction (13) as $\mathcal{T}_*(\eta) = \mathcal{D}\mathcal{T}(\eta)$.

A little reflection will make it plain that the direct radiation is given by

$$\mathcal{T}_{\odot}(\eta) = t_u^{\odot}[(1-\eta)t_c^{\odot} + \eta t_\Delta^{\odot}]t_l^{\odot}, \tag{53}$$

with $t_x^{\odot} = \exp[-\tau_{xb}/\mu_x^+]$. Then $\mathcal{T}_*(\eta) = \mathcal{T}(\eta) - \mathcal{T}_{\odot}(\eta)$.

Let us now consider the simplified model atmosphere consisting of both air that does not scatter radiation at all ($a_u = a_\Delta = a_l = 0$) and perfectly scattering clouds ($r_c + t_c = 1$). Its diffuse transmissivity may be shown to be expressible as

$$\mathcal{T}_*(\eta) = t_u^{\odot} t_l^{\odot}(1-\eta)\{t_c^* + r_s t_l^{\odot 2}(1-t_c)[t_c(1-\eta) + t_\Delta^{\odot}\eta]\}, \tag{54}$$

where by Eq. (49) and $t_c^{\odot} = e^{-\tau_{cb}/\mu_c^+}$,

$$t_c^* = \frac{1}{1+(1-f_c)\tau_{cb}/\mu_c^+} - e^{-\tau_{cb}/\mu_c^+}. \tag{55}$$

The singular case $f_c = 1$ leads to $t_c = 1$ and hence $r_c = 0$, which is equivalent to $a_c = 0$, i.e. $\tau_{cb} = 0$ and thence $t_c^{\odot} = 1$ as well as $t_c^* = 0$.

From (54) the diffuse transmissivity under overcast conditions follows at once

$$\mathcal{T}_*(0) = t_u^{\odot} t_l^{\odot}[t_c^* + r_s t_l^{\odot 2}(1-t_c)t_c] \tag{56}$$

while that for cloudless skies is simply

$$\mathcal{T}_*(1) = 0,$$

as we expect it to be.

With an overcast sky, there are two extreme values for which the diffuse transmissivity (56) vanishes: $t_c = 1$ (the singular case described before, amounting to a cloud layer with "invisible clouds") and $t_c = 0$ (optically deep clouds, $\tau_{cb} \longrightarrow \infty$).

For all values $\eta < 1$, $\mathcal{T}_*(\eta)$ has a maximum for a certain value of τ_{cb}, say $\hat{\tau}_{cb}$. The diffusive transmissivity first increases with the optical thickness of the cloud, up to the value $\hat{\tau}_{cb}$, thence slowly decreases for $\tau_{cb} > \hat{\tau}_{cb}$.

If the ground albedo is low or vanishes, the diffuse transmissivity simplifies to

$$\mathcal{T}_*(\eta) = t_u^{\odot} t_l^{\odot}(1-\eta)t_c^*, \tag{57}$$

and under overcast skies $\mathcal{T}_*(0) = t_u^{\odot} t_l^{\odot} t_c^*$. Clearly, the diffuse transmissivity over a reflecting ground (or sea) is greater than over a black surface, unless the cloud layer is very deep.

If instead of non-scattering air overlying a reflecting surface we consider (besides the scattering clouds) a scattering lower atmospheric layer resting on a black surface, we would get the following diffuse transmissivity ($r_u = r_\Delta = r_s = 0; r_l \neq 0$):

$$\begin{aligned}\mathcal{T}_*(\eta) = t_u^{\odot}\{&[t_c t_l(1-\eta) + t_\Delta^{\odot} t_l^* \eta][1 + r_c r_l(1-\eta)] \\ &+ t_l^{\odot}(1-\eta)(t_\Delta^{\odot} r_c r_l \eta - t_c^{\odot})\}.\end{aligned} \tag{58}$$

For overcast skies, this yields $\mathcal{T}_*(0) = t_u^{\odot}[t_c t_l(1 + r_c r_l) - t_l^{\odot} t_c^{\odot}]$, while for clear skies we get $\mathcal{T}_*(1) = t_u^{\odot} t_\Delta^{\odot} t_l^* > 0$, the latter positive contribution being due to the scattering in the lower atmosphere.

Although we could examine the general case (for $r_x \neq 0$, $x \in \{\Delta, l, s, u\}$), we will forgo the resulting lengthy formula for the diffuse transmissivity, as no new qualitative inferences will follow from it.

4. Discussion

The literature on the Ångström-Prescott and similar equations has been reviewed by **Akinoglu** (2008), whom I will be citing below. To avoid the clumsy repetition of the publication year in parenthesis, in this section I shall omit it on the understanding that his survey is always being referred to.

Akinoglu states that the Ångström coefficients a and b "have been expressed in terms of different geographical and climatic parameters such as latitude, altitude, sunshine fraction", and he concludes that those coefficients "depend on all physical, spatial and the dynamic properties of the atmosphere at the region of interest". Our model makes this conclusion compelling, because the different transmissivities are quantities certainly varying "dynamically" from one place to another (geographically).

After a short review of earlier attempts, **Akinoglu** presents a model of his own that produces a quadratic expression in the fractional sunshine duration, with corresponding coefficients, the first of which in our notation reads $a_0 = t_u t_\Delta t_l t_c(1 + r_s r_c)$ (the quantities being effective ones, not dependent upon wavelength). Let us compare this expression with what results from (40) when we assume $r_u = r_l = 0$. Then, in view of Eqs. (50) and (52), we get $\mathcal{T}(0) = a_0 = t_u t_c t_l(1 + r_c r_s t_l^2)$. If furthermore we assume that the lower atmosphere does not absorb solar radiation, then $t_l = 1$ and $a_0 = t_u t_c(1 + r_c r_s)$, which can finally be made to coincide with **Akinoglu**'s expression if we also require the interstices of the middle atmosphere to be transparent (i.e., $t_\Delta = 1$). The other two coefficients proposed by **Akinoglu** are more difficult to compare, because they involve "the atmospheric forward scattering coefficient", as well as "the total atmospheric back-scattering", which seem to be what we call reflectivities. The latter "can be defined with two components" (and seemingly corresponds to our r_n). It seems to me that if **Akinoglu**'s "forward scattering coefficient" is set to zero (being already accounted for in the reflectivities of our model), and if his $t_\Delta t_c$ is always properly read to mean only t_c, then the coefficients can be made to agree! But however well the coefficients he proposes and those in our work might eventually compare, I cannot help calling his "derivation" at best a felicitous *ad hoc* attempt to confer a definite meaning to the coefficients of polynomial regression.

In discussing the merits of a quadratic regression between transparency and relative sunshine duration, **Akinoglu** suggests three reasons for "non-linearity", among which one is a conjecture based on his model of the physical meaning of the three coefficients in the quadratic regression (his a_0 quoted above being among them): "Finally, back-scatter effects may lead to a non-linear term [in] the relation between $\mathcal{T}$ and η...". From our standpoint, this is again a compelling conclusion, since a necessary condition for a quadratic formula relating the quantities in question is that the surface be non-black ($r_s \neq 0$), i.e. that there exist "back-scatter effects".

Akinoglu cites evidence showing that the quadratic form fits daily data of two locations in Turkey better than does the linear Ångström-Prescott formula. He gives numerical values for the coefficients of the quadratic regression, first for daily and then for monthly averages, those of the latter being $a_0 = 0.195$, $a_1 = 0.676$, and $a_2 = -0.142$. Appealing to expression (40) when $r_u = 0$ (for then the quadratic form arises naturally in our scheme), we obtain from (36) for the third coefficient $a_2 = t_u(t_\Delta - t_c)t_l(1 - r_s r_l)(r_\Delta - r_c)A_l$, which under normal conditions we expect to be negative, since $t_\Delta - t_c > 0$, $r_\Delta - r_c < 0$. The same conclusion is arrived at if $r_u = r_l = 0$, bearing out **Akinoglu**'s quoted signs for a_2.

Şahin and Şen (2008) point out that the coefficients of the linear or quadratic transparency are not constant in time. **Akinoglu** states that they have values depending on the averaging interval. We may say that, once more, their conclusions are by no means surprising against the background of our present model.

Conclusion

Modeling surface irradiation, by relating it to the widely measured sunshine at many places of the world through the simple linear Ångström-Prescott regression formula, has a respectable history. The need for an extension to include quadratic or even third-order polynomial regression has been increasingly felt in recent years. Many statistical analyses have been carried out on this basis, and many more will follow, including innovative approaches to parameter estimation (**Badescu** 2008).

However, the attempts to furnish a physical interpretation of the regression coefficients have been largely *ad hoc*. Here, instead, we submit a rationale for such models, based on a two-stream approximation of the vertical fluxes crossing an atmosphere fashioned from three characteristic layers, an approximation which in turn rests firmly on the phenomenological theory of radiative transfer through planetary atmospheres.

For years to come, simple and even simplistic models will be needed in order to cope with the practical issues regarding the mutual relation between insolation and relative sunshine duration at sites suffering from a lack of spatial and temporal coverage. A definite conclusion that we draw from this work is that the classic linear Ångström-Prescott formula cannot be expected to be a good regression equation when air scattering and ground reflectivity are *not* negligible. In these cases, at least a quadratic formula between the "clearness index" and the relative sunshine is required; a still better regression when scattering prevails ought to be a third-order polynomial. Even more, a polynomial of higher order would be called for if we wished to infer the meteorological parameters of the model by solving what is known as the inverse problem.

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